Optimization in the Context of Active Control of Sound*

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Abstract. A problem of eliminating the unwanted time-harmonic noise on a predetermined region of interest is solved by active means, i.e., by introducing the additional sources of sound, called controls, that generate the appropriate annihilating signal (anti-sound). The general solution for controls has been obtained previously for both the continuous and discrete formulation of the problem. Next, the control sources are optimized using different criteria. Minimization of the overall absolute acoustic source strength is equivalent to minimization of multi-variable complex functions in the sense of L_1 with conical constraints. The global L_1 optimum appears to be a special layer of monopoles on the perimeter of the protected region. The use of quadratic cost functions, e.g., the L_2 norm of the controls, leads to a versatile numerical procedure. It allows one to analyze sophisticated geometries in the case of a constrained minimization. Finally, minimization of power consumed by an active control system always involves interaction between the sources of sound and the surrounding acoustic field, which was not the case for either L_1 or L_2. One can, in fact, build a control system that would require no power input for operation and may even produce a net power gain while providing the exact noise cancellation. This, of course, comes at the expense of having the original sources of noise produce even more energy.

1 Introduction

Let $\Omega \subset \mathbb{R}^n$ be a given domain (bounded or unbounded), and Γ be its boundary: $\Gamma = \partial\Omega$, where the dimension of the space n is either 2 or 3. Both on Ω and on its complement $\Omega_1 = \mathbb{R}^n \backslash \Omega$ we consider the time-harmonic acoustic field $u = u(\boldsymbol{x})$, $\boldsymbol{x} \in \mathbb{R}^n$, governed by the inhomogeneous Helmholtz equation:

$$\boldsymbol{L}u \equiv \Delta u + k^2 u = f, \tag{1}$$

subject to the Sommerfeld radiation boundary conditions at infinity:

$$u(\boldsymbol{x}) = O\left(|\boldsymbol{x}|^{-\frac{n-1}{2}}\right), \quad \frac{\partial u(\boldsymbol{x})}{\partial|\boldsymbol{x}|} + iku(\boldsymbol{x}) = o\left(|\boldsymbol{x}|^{-\frac{n-1}{2}}\right), \quad \text{as} \quad |x| \longrightarrow \infty. \tag{2}$$

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Boundary conditions (2) specify the direction of wave propagation, and distinguish between the incoming and outgoing waves at infinity by prescribing the outgoing direction only; they guarantee the unique solvability of the Helmholtz equation (1) for any compactly supported right-hand side $f = f(\boldsymbol{x})$. Note that all the solutions considered hereafter represent traveling waves and are generally speaking complex-valued.

The source terms $f = f(\boldsymbol{x})$ in equation (1) can be located on both Ω and its complement $\Omega_1 = \mathbb{R}^n \backslash \Omega$; to emphasize the distinction, we denote

$$f = f^+ + f^-, \qquad \operatorname{supp} f^+ \subset \Omega, \quad \operatorname{supp} f^- \subset \Omega_1. \tag{3}$$

Accordingly, the overall acoustic field $u = u(\boldsymbol{x})$ can be represented as a sum of the two components:

$$u = u^+ + u^-, \tag{4}$$

where u^+ is driven by the interior sources f^+, and u^- is driven by the exterior sources f^- w.r.t. Ω:

$$\boldsymbol{L}u^+ = f^+, \tag{5a}$$

$$\boldsymbol{L}u^- = f^-. \tag{5b}$$

Note, both $u^+ = u^+(\boldsymbol{x})$ and $u^- = u^-(\boldsymbol{x})$ are defined on the entire $\mathbb{R}^n$, the superscripts "+" and "−" refer to the sources that drive each of the field components rather than to the domains of these components. The setup described above is schematically shown in Figure 1 for the case of a bounded domain Ω.

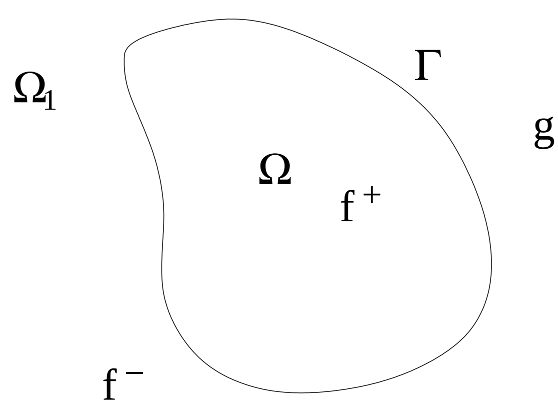


Fig. 1. Geometric setup.

Hereafter, we will call the component u^+ of (4), (5a) *sound*, or "friendly" part of the total acoustic field; the component u^- of (4), (5b) will accordingly be called *noise*, or "adverse" part of the total acoustic field. Ω will be a predetermined region of space to be protected from noise. This means that we would like to eliminate the noise component of $u(\boldsymbol{x})$ inside Ω, while leaving the sound component there unaltered. In the mathematical framework that we have adopted, the component u^- of the total acoustic field, i.e., the response to the adverse sources f^- [see (3), (4), (5)], will have to be canceled out on Ω, whereas the component

u^+, i.e., the response to the friendly sources f^+, will have to be left unaffected on Ω. A physically more involved but conceptually easy to understand example is that inside the passenger compartment of an aircraft we would like to eliminate the noise coming from the propulsion system located outside the fuselage, while not interfering with the ability of the passengers to listen to the inflight entertainment programs or simply converse. Another good example is found in medicine, where high levels of periodic noise are produced by resonance coils in magnetic resonance imaging (MRI) machines.

The concept of *active noise control* implies that the component u^- is to be suppressed on Ω by introducing additional sources of sound $g = g(\boldsymbol{x})$ exterior with respect to Ω, $\operatorname{supp} g \subset \Omega_1$, so that the total acoustic field $\tilde{u} = \tilde{u}(\boldsymbol{x})$ be now governed by the equation [cf. formulae (1), (3)]:

$$\boldsymbol{L}\tilde{u} = f^+ + f^- + g, \tag{6}$$

and coincide with only the friendly component u^+ on the domain Ω:

$$\tilde{u}\big|_{\boldsymbol{x}\in\Omega} = u^+\big|_{\boldsymbol{x}\in\Omega}. \tag{7}$$

The new sources $g = g(\boldsymbol{x})$ of (6), see Figure 1, will hereafter be referred to as the *control sources* or simply *controls.* An obvious solution for these control sources is $g = -f^-$. This solution, however, is clearly sub-optimal because on one hand, it requires an explicit and detailed knowledge of the structure and location of the sources f^-, which is, in fact, superfluous, see [1]. On the other hand, its implementation in many cases, like in the previously mentioned example with an airplane, may not be feasible. Fortunately, there are other solutions of the foregoing noise control problem (see Section 2 and [1] for detail), and some of them may be preferable from both the theoretical and practical standpoint.

The area of active control of sound has a rich history of development, both as a chapter of theoretical acoustics, and in the perspective of many different applications. We refer the reader to the monographs [2–4] that, among other things, contain a detailed survey of the literature. Potential applications range from the aircraft industry to manufacturing industry to ground and air transportation to the military to consumer products and other fields, including even such highly specialized and narrow areas as acoustic measurements in the wind tunnels. It is generally known that active techniques are more efficient for lower frequencies, and they are usually expected to complement passive strategies (sound insulation, barriers, etc.) that are more efficient for higher frequencies, because the rate of sound dissipation is proportional to the square of the frequency [5].

In the current paper we only analyze the constant-coefficient Helmholtz equation (1), which governs the field throughout the entire $\mathbb{R}^n$. This is most straightforward formulation. However, one can as well consider other, more complex, cases that involve variable coefficients, different types of far-field behavior, discontinuities in the material properties, and maybe even nonlinearities in the governing equations over some regions. Approaches to obtaining solutions for active controls in these cases are based on the theory of generalized Calderon's potentials and boundary projections, and can be found in our previous paper [1] and in the monograph by Ryaben'kii [6, Part VIII].

2 General Solutions for Control Sources

A general solution for the volumetric continuous control sources $g = g(\boldsymbol{x})$ is given by the following formula ($\Omega_1 = \mathbb{R}^n \backslash \Omega$):

$$g(\boldsymbol{x}) = -\boldsymbol{L}w\big|_{\boldsymbol{x}\in\Omega_1}, \tag{8}$$

where $w = w(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega_1$, is a special auxiliary function-parameter that parameterizes the family of controls (8). The function $w(\boldsymbol{x})$ must satisfy the Sommerfeld boundary conditions at infinity (2), and at the interface Γ the function w and its normal derivative have to coincide with the corresponding quantities that pertain to the total acoustic field u given by formula (4):

$$w\big|_{\Gamma} = u\big|_{\Gamma}, \qquad \frac{\partial w}{\partial \boldsymbol{n}}\bigg|_{\Gamma} = \frac{\partial u}{\partial \boldsymbol{n}}\bigg|_{\Gamma}. \tag{9}$$

Other than that, the function $w(\boldsymbol{x})$ used in (8) is arbitrary, and consequently formula (8) defines a large family of control sources, which provides ample room for optimization. The justification for formula (8) as general solution for controls can be found in [1]. In [7], we also emphasize that the controls

$$g(\boldsymbol{x}) = \int g(\boldsymbol{y})\delta(\boldsymbol{x} - \boldsymbol{y})d\boldsymbol{y} = g * \delta$$

given by (8) are *actually volumetric control sources of the monopole type* with regular density $g \in L_1^{(\mathrm{loc})}(\mathbb{R}^n)$ [assuming that $w(\boldsymbol{x})$ was chosen sufficiently smooth so that to guarantee local absolute integrability of $g(\boldsymbol{x})$].

Note that to obtain the controls (8) one needs no knowledge of the actual exterior sources of noise f^-. All one needs to know is u and $\frac{\partial u}{\partial \boldsymbol{n}}$ on the perimeter Γ of the protected region Ω. In a practical setting, $u\big|_{\Gamma}$ and $\frac{\partial u}{\partial \boldsymbol{n}}\big|_{\Gamma}$ can be interpreted as measurable quantities that are supplied to the control system as the input data. Moreover, these measurable quantities can refer to the overall acoustic field u, rather than only its unwanted component u^-. In other words, the methodology can automatically distinguish between the signals coming from the exterior and interior sources, and can tune the controls so that they cancel only the unwanted exterior signal. This capability, which essentially implies that the control sources (8) are insensitive to the interior sound $u^+(\boldsymbol{x})$, is extremely important because in many applications the overall acoustic field always contains a component that needs to be suppressed along with the part that needs to be left intact.

Along with the volumetric controls (8), one can also consider *surface controls,* i.e., the control sources that are concentrated only on the interface Γ. A general solution for the surface controls is given by (see [8, 7]):

$$g^{(\mathrm{surf})} = -\left[\frac{\partial w}{\partial \boldsymbol{n}} - \frac{\partial u}{\partial \boldsymbol{n}}\right]_{\Gamma} \delta(\Gamma) - \frac{\partial}{\partial \boldsymbol{n}}\big([w-u]_{\Gamma}\delta(\Gamma)\big), \tag{10}$$

where $w = w(\boldsymbol{x})$, as before, denotes the auxiliary function-parameter. In contradistinction to the previous case, now it has to satisfy the homogeneous

Helmholtz equation on the complementary domain: $\boldsymbol{L}w = 0$ for $\boldsymbol{x} \in \Omega_1$, and the Sommerfeld boundary condition at infinity (2), but at the interface Γ it may be arbitrary, i.e., it does not have to meet boundary conditions (9). The corresponding discontinuities [expressions in rectangular brackets in formula (10)] drive the surface control sources. The first term on the right-hand side of (10) represents the density of a single-layer potential, which is a layer of monopoles on the interface Γ, and the second term on the right-hand side of (10) represents the density of a double-layer potential, which is a layer of dipoles on the interface Γ. The fundamental properties of the surface controls (10) are the same as those of the volumetric controls (8) — they are also insensitive to the interior sound $u^+(\boldsymbol{x})$, and do not require any knowledge of the actual sources of noise f^-.

In the family of surface controls (10) we identify two important particular cases. First, the cancellation of $u(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega$, can be achieved by using surface monopoles only, i.e., by employing only a single-layer potential as the anti-sound. To do that, we need to find $w(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega_1$, such that there will be no discontinuity on Γ between $u(\boldsymbol{x})$ and $w(\boldsymbol{x})$, i.e., in the function itself, and the discontinuity may only "reside" in the normal derivative [see formula (10)]. This $w(\boldsymbol{x})$ will obviously be a solution of the following external Dirichlet problem:

$$\begin{gathered} \boldsymbol{L}w = 0, \quad \boldsymbol{x} \in \Omega_1, \\ w\big|_{\Gamma} = u\big|_{\Gamma}, \end{gathered} \tag{11}$$

subject to the Sommerfeld boundary conditions (2). Problem (11) is always uniquely solvable on $\Omega_1 = \mathbb{R}^n \backslash \Omega$. Second, one can employ only the double-layer potential to cancel out $u(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega$, i.e., use only surface dipoles as the control sources. In this case, the function $w(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega_1$, has to be chosen such that the discontinuity on Γ be only in the function itself, i.e., between the actual values of $u(\boldsymbol{x})$ and $w(\boldsymbol{x})$, and not between the normal derivatives. This $w(\boldsymbol{x})$ should then solve the following external Neumann problem:

$$\begin{gathered} \boldsymbol{L}w = 0, \quad \boldsymbol{x} \in \Omega_1, \\ \frac{\partial w}{\partial \boldsymbol{n}}\bigg|_{\Gamma} = \frac{\partial u}{\partial \boldsymbol{n}}\bigg|_{\Gamma}, \end{gathered} \tag{12}$$

again, subject to the Sommerfeld conditions at infinity (2) that guarantee the solvability of (12). We therefore see that surface control sources (10) are basically given by combinations of the monopole and dipole layers, with the two "extreme" cases corresponding to either only monopoles, see (11), or only dipoles, see (12).

Altogether, we have now introduced active controls of two different types on the surface, but only one type of the volumetric controls — monopoles. From the standpoint of physics, the monopole and dipole sources provide different types of acoustic excitation. A point monopole source can be interpreted as a vanishingly small pulsating sphere with isotropic radiation, whereas a dipole source resembles a small oscillating membrane that has a particular directivity of radiation. In the genuine time-dependent context, monopole sources alter the balance of mass in the system; they are scalar in nature and reside on the right-hand side of the continuity equation. Dipole sources alter the balance of force, they are vectors and reside on the right-hand side of the momentum equation, see [7]. This distinction warrants a separate consideration of the monopole and dipole type sources for the point-wise or surface excitation. However, for the time-harmonic volumetric excitation a separate consideration of dipole fields appears superfluous because any volumetric distribution of dipoles can, in fact, be recast in the form of an equivalent volumetric distribution of monopoles, see [7]. In so doing, the Helmholtz equation (1) is re-written as follows:

$$\Delta u(\boldsymbol{x}) + k^2 u(\boldsymbol{x}) = \operatorname{div} \boldsymbol{b}_{\mathrm{vol}}(\boldsymbol{x}) + i\omega\rho_0 q_{\mathrm{vol}}(\boldsymbol{x}), \tag{13}$$

where *the volume velocity per unit volume* q_{vol}, also known as *acoustic source density,* represents monopoles, *the force per unit volume* $\boldsymbol{b}_{\mathrm{vol}}$ represents dipoles, see [2, 9]; the wavenumber k is given by $k = \omega/c$, where ω is the frequency of the temporal oscillations, and $u = u(\boldsymbol{x})$ has the meaning of acoustic pressure.

A similar discrete formulation of the noise control problem has been previously developed in the context of finite differences, see [8, 7, 10, 11, 6]. We use it hereafter for computations of Sections 3.1 and 3.2.

3 Optimization of the Control Sources

Once the general solution for controls (8) or (10) is available, the next step is to decide what particular control distribution will be optimal for a specific setting. There are many possible criteria for optimality that one can use. In practical problems the cancellation of noise is often only approximate, and the key criterion for optimization is the quality of this cancellation. In contradistinction to that, here we are considering ideal, or exact, cancellation, i.e., every control field (8) or (10) completely eliminates the unwanted noise on Ω. Consequently, the criteria for optimality that we can employ will not include the level of the residual noise and should rather depend only on the control sources themselves.

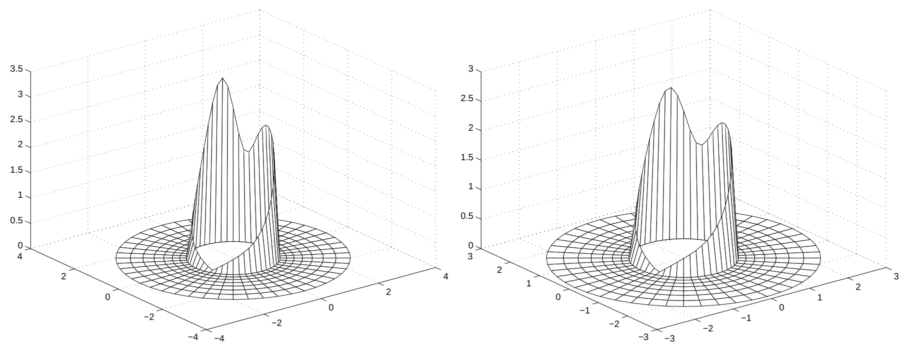
3.1 Optimization in the Sense of L_1

Clearly, the physical meaning of the control sources will be the same as that of the original sources, see equations (1), (6), and (13). In this perspective, we would first argue for selecting the optimal control sources based on minimization of their *overall absolute acoustic source strength.* Mathematically, this translates into minimization of the L_1 norm of the control sources:

$$\|g\|_1 \equiv \int\limits_{\operatorname{supp} g} |g(\boldsymbol{x})| d\boldsymbol{x} \longrightarrow \min, \tag{14}$$

where the search space for minimization in (14) includes all the appropriate auxiliary functions $w(\boldsymbol{x})$, by means of which the controls $g(\boldsymbol{x})$ are defined, see (8), (9). The advantage of using this criterion for optimization is that it has *a clear physical interpretation,* and the quantities involved, namely, the volume velocity and the force applied to fluid particles (14), actually characterize the corresponding engineering devices (i.e., actuators in the active noise control system).

The disadvantage of using the L_1 norm of the control sources as a cost function for optimization is that the discrete counterpart of minimization problem (14) is very difficult to solve numerically, see [7]. This discrete problem appears non-smooth and non-linear; moreover, it is only "marginally" convex as it essentially reduces to optimization over a large number of cones. Linear programming does not apply to this problem because of the complex nature of the quantities involved. And even the most sophisticated nonlinear programming techniques known as interior point methods can only solve such problems for very low dimensions. Our best numerical results were obtained with the software package SeDuMi by J. F. Sturm [12].



(a) L_1 optimal solution (b) Surface monopoles

Fig. 2. Computed control sources.

In Figure 2, we compare the computed L_1 optimum with surface monopoles defined by (10), (11), for a particular setup when the protected region Ω is a disk. There are no visual differences between the two solutions. In fact, this behavior appears coherent and has been observed for all other cases computed in [7]. Moreover, the fact of coincidence of the L_1 minimum (14) with surface monopoles (10), (11) has been corroborated in [7] by the grid convergence tests.

Motivated by these consistent numerical observations, we have also been able to rigorously prove that *the global minimum* of the control sources (8), (9) in the sense of L_1 is given by the surface monopoles (10), (11). The proof obtained in [7] holds for both the continuous and discrete formulation of the problem, but only *in the one-dimensional case.* Even though we have not yet been able to prove a similar result for a general multi-dimensional framework, we still believe that it holds, because a combination of the two-dimensional numerical evidence and a one-dimensional accurate proof cannot, in our opinion, be a mere coincidence. As such, we put forward the following

Conjecture 1. Let a complex function $w = w(\boldsymbol{x})$ be sufficiently smooth on Ω_1 so that the operator $\boldsymbol{L}$ of (1) can be applied to $w(\boldsymbol{x})$ in the classical sense, and $\boldsymbol{L}w \in L_1^{(\mathrm{loc})}(\Omega_1)$. Let also $w(\boldsymbol{x})$ satisfy the interface conditions (9) and the Sommerfeld conditions (2). Then, the greatest lower bound for the L_1 norms of all the control sources $g(\boldsymbol{x})$ obtained with such auxiliary functions $w(\boldsymbol{x})$ using (8), is given by the L_1 norm on Γ of the magnitude of surface monopoles $\nu(\boldsymbol{x}) \equiv -\left[\frac{\partial w}{\partial \boldsymbol{n}} - \frac{\partial u}{\partial \boldsymbol{n}}\right]_{\boldsymbol{x}\in\Gamma}$, see (10), for a particular $w(\boldsymbol{x})$ that solves problem (11):

$$\inf_{w(\boldsymbol{x})} \int_{\Omega_1} |g(\boldsymbol{x})| d\boldsymbol{x} = \int_{\Gamma} |\nu(\boldsymbol{x})| ds. \tag{15}$$

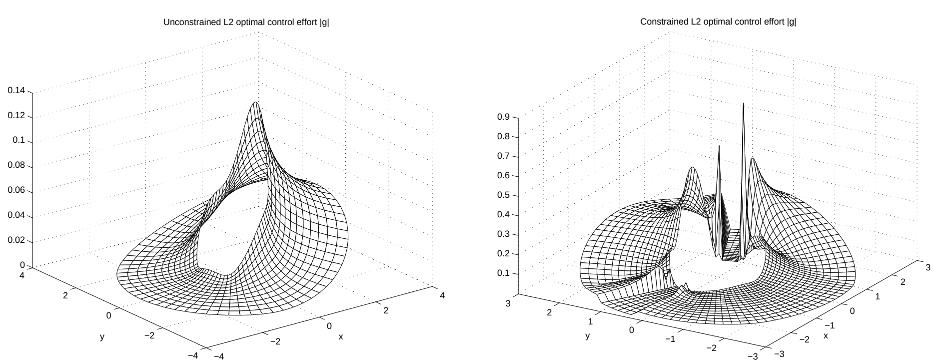
Conjecture 1 implies, in particular, that no numerical optimization is needed for determining what the L_1-optimal active controls are.

3.2 Optimization in the Sense of L_2

A natural quadratic criterion for optimization is the L_2 norm of the controls:

$$\|g\|_2 \equiv \sqrt{\int_{\mathrm{supp}\, g} |g(\boldsymbol{x})|^2 d\boldsymbol{x}} \longrightarrow \min. \tag{16}$$

The optimum according to (16) is *easy to compute numerically,* especially in the unconstrained case. However, the quantity $\|g\|_2$ does not have a clear physical meaning, such as $\|g\|_1$ of Section 3.1, which is the absolute acoustic source strength. We emphasize that $\|g\|_2$ is the L_2 norm of the residuals, and not of the solution itself, which is often related to energy.



(a) Unconstrained case (b) Constrained case

Fig. 3. Computed L_2 optimal control sources.

Nonetheless, in [13] we have computed several L_2 optima. In the unconstrained case we have been able to prove in [13] that the matrix of the system which is actually solved in the sense of the least squares has full column rank and consequently, no Moore-Penrose type arguments are required. An example of the unconstrained L_2-optimal solution for Ω being a disk is shown in Figure 3(a). We have also shown in [13] that our unconstrained discrete L_2-optimal solutions do converge to the solutions obtained previously in [1] by a semi-analytic spectral type method for those case, for which the latter are available.

An example of the L_2-optimal solution with constraints of equality type is shown in Figure 3(b). We have required that no control effort can be present in two sector-shape regions that can be thought of as portholes in the cylindrical fuselage. This example shows that by optimizing in the sense of L_2 one can analyze relatively complex geometries. Of course, the L_2-optimal solutions are distinctly different from the L_1-optimal solutions of Section 3.1. They tend to spread over the volume rather than concentrate on the interface, and they do not reduce to any known special cases, such as surface monopoles.

3.3 Optimization of Power

A quadratic optimization criterion that, unlike $\|g\|_2$ of Section 3.2, would have a transparent physical interpretation, is energy or power. In acoustics, using this criterion will necessarily involve the interaction between the sources of sound and the surrounding field. It turns out [14] that active noise control can extract more than enough power from the acoustic field to achieve exact noise cancellation. In fact, one can control interior noise, and at the same time increase acoustic loading on exterior noise sources. This increases emitted noise power and allows the noise control to extract even more power from the field.

Let the perimeter Γ, where surface controls (10) are defined, be closely surrounded by contours Γ^+ and Γ^-. Introducing the velocity potential [5] to express velocity in terms of pressure p and averaging over the period $T = 2\pi/\omega$, we seek to minimize the combined control power requirements at frequencies ω and $-\omega$ (equation (1) is obtained by Fourier transforming the wave equation w.r.t. time):

$$W = \frac{2}{\rho\omega} \oint_{\Gamma^+ \bigcup \Gamma^-} \operatorname{Im}\left(p \frac{\partial \overline{p}}{\partial \boldsymbol{n}} \right) da \longrightarrow \min . \tag{17}$$

Exact noise cancellation implies that along Γ^+ (just inside Γ) pressures and velocities coincide with the sound field produced by interior sources only, so at best we can collect all of the power leaving the interior. However, we have the freedom to alter the exterior acoustic field so that more power can be extracted. Cylindrical geometry (Ω being a disk of radius R) and Fourier transform in the circumferential direction allow us to carry out optimization (17) analytically.

For the noise field $p_m^-(r)$ at the Fourier circumferential mode m, the optimal total pressure (that guarantees minimum power) just outside Γ is given by [14]:

$$p_m(r) = \frac{p_m^-(R)}{2J_m(kR)} \left(J_m(kr) + iY_m(kr) \right),$$

and the minimal control power due to only p^- is

$$W_m = -\frac{2|p_m^-(R)|^2}{\omega\rho [J_m(kR)]^2} \leq 0.$$

When $p_m^-(r)$ represents the field from a unit point source located at $r = s > R$, the control power requirements for mode m due to this source evaluate to

$$W_m^s = -\frac{[J_m(ks)]^2 + [Y_m(ks)]^2}{8\omega\rho} < 0. \tag{18}$$

The magnitudes of W_m^s of (18) increase rapidly with m. Therefore, the Parseval relation implies that more and more power can be extracted from the noise field when the number of circumferential modes taken into account grows. The physical explanation is that the optimal control increases acoustic loading seen by the noise sources, thereby increasing their power output. In a practical setting, power extraction is limited by the power available at the noise sources and by engineering constraints. A reasonable approach would be to optimize control power for $|m| \le M$ but for $|m| > M$ leave the exterior field unaltered. In all cases, the control sources along Γ are given by formula (10), in which the difference $[w - u]$ shall be interpreted as the optimal pressure field due to controls.

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