The Doppler Effect for SAR

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Abstract

We present a mathematical analysis of the start-stop approximation that is routinely used in synthetic aperture radar (SAR) imaging. The objective is to quantify the effect of those factors that the start-stop approximation neglects. They include the displacement of the antenna during the pulse round-trip time between the radar platform and the target and the Doppler frequency shift. We show that both phenomena can be accounted for by appropriately correcting the signal processing algorithm. This, in turn, requires computing the emitted and scattered field with the help of the Lorentz transform. If the correction is done, then the effect of the antenna motion on the image becomes negligibly small. Otherwise, the image gets shifted and also distorted. For some imaging settings, the distortions due to the start-stop approximation may become substantial, which is not commonly discussed in the SAR literature.

Keywords: synthetic aperture radar (SAR), start-stop approximation, Lorentz transform, correction for antenna motion.

We analyze the Doppler effect in the context of synthetic aperture radar (SAR) imaging, following up and expanding on our earlier results [1]. Understanding of the Doppler effect is critical for evaluating and then mitigating the effect of the start-stop approximation on the image. The start-stop approximation is a common tool in SAR signal processing. It simplifies the analysis by assuming that the radar antenna is motionless during the transmission and reception of the interrogating signals.

The two important effects ignored under the start-stop approximation are the displacement of the antenna during the time the signal travels back and forth between the antenna and the Earth's surface and the Doppler frequency shift. The latter appears because the antenna actually moves when the pulse is emitted and the reflected signal received. Our main objective is to provide a quantitative analysis of the impact of these two effects on SAR imaging.

The role of the start-stop approximation can be analyzed by having the standard retarded potential

$$P\left(t-\frac{2r}{c}\right)$$

replaced with the new propagator

$$P\left(t\left(1+2\frac{\mathfrak{v}}{c}\cos\gamma\right)-\frac{2r}{c}\left(1+\frac{\mathfrak{v}}{c}\cos\gamma\right)\right) \quad (1)$$

derived using the Lorentz transform, which preserves the governing wave equation in the case of moving transmitters/receivers. Formula (1) takes into account both the frequency shift and antenna displacement. The quantity $\mathfrak{v}$ in (1) is the platform velocity, γ is the angle between the velocity and the direction to the target, and $2r$ is the round-trip distance, see Figure 1.

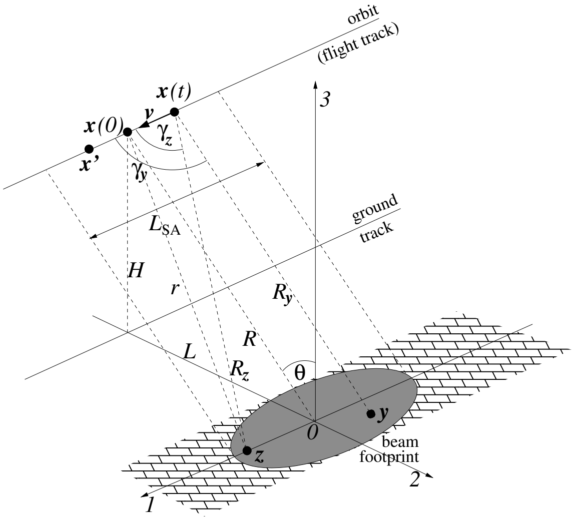

Figure 1: Monostatic stripmap SAR.

The new propagator (1) helps one correct the SAR signal processing algorithm and compensate for the adverse effect of the start-stop approximation on the image. In doing so, implementing the correction is relatively easy because it does not require any additional information besides the geometric quantities (angles and distances, see Figure 1) and the platform velocity $\mathfrak{v}$. This is in contradistinction, say, to the corrections aimed at mitigating the distortions due to the Earth's ionosphere, see [2] and [3, Chapter 3], for which one first needs to reconstruct the unknown characteristics of the ionosphere.

Our analysis is based on representing the image as a convolution of the ground reflectivity $\nu(\boldsymbol{z})$ that characterizes the target with the imaging kernel $W(\boldsymbol{y},\boldsymbol{z}) = W(\boldsymbol{y}-\boldsymbol{z})$ that characterizes the radar system:

$$I(\boldsymbol{y}) = \int \nu(\boldsymbol{z}) W(\boldsymbol{y},\boldsymbol{z}) d\boldsymbol{z}. \qquad (2)$$

The effect of the Doppler correction is quantified by studying the properties of the corresponding kernel W. Representation (2) allows for a rigorous consideration of all the effects and robust prediction of the system performance.

If, on the other hand, the SAR signal processing procedure is not corrected to account for the motion of the radar platform, then the resulting SAR image becomes shifted in the azimuthal direction and may also be distorted (blurred). In many cases, the foregoing distortions will be small. Yet for certain imaging scenarios, the distortions of the image caused by the start-stop approximation may appear significant. The type of SAR imaging systems that may be particularly prone to this kind of distortions are those that exploit the interrogating waveforms (chirps) with low rate of frequency modulation, i.e., the so-called frequency modulated continuous waves (FMCW). These systems are actually contemplated and built in practice (see, e.g., [4, 5], as well as the new ViSAR project by DARPA), because the FMCW waveforms seem to present fewer hardware limitations for airborne or spaceborne SAR platforms. At the same time, to the best of our knowledge the possibility of image deterioration caused by the start-stop approximation did not receive a proper attention in the SAR literature, with the exception of [3, Chapter 6]. Other relevant publications include [6].

Let us also note that some existing books on SAR, e.g., [7, 8], treat the Doppler effect with notable inaccuracies, erroneously attributing to it the mechanism of azimuthal resolution. Hence, we find it important to clearly demonstrate the role of both the actual physical Doppler effect, which is due to the antenna velocity $\mathfrak{v}$ and is referred to as the Doppler effect in fast time, as well as that of the so-called Doppler effect in slow time. The latter is a linear variation of the local wavenumber along the synthetic array, and it is this phenomenon that enables the signal compression in the azimuthal direction and yields the azimuthal resolution.

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References

[1] S. V. Tsynkov. *SIAM J. on Imaging Sciences*, 2(2):646–669, 2009.

[2] Mikhail Gilman, Erick Smith, and Semyon Tsynkov. *Inverse Problems*, 29(5):054005 (35pp), April 2013.

[3] Mikhail Gilman, Erick Smith, and Semyon Tsynkov. *Transionospheric Synthetic Aperture Imaging*. Birkhäuser, New York, 2017. (To appear).

[4] A. Meta, P. Hoogeboom, and L. P. Ligthart. *IEEE Transactions on Geoscience and Remote Sensing*, 45(11):3519–3532, Nov 2007.

[5] Angel Ribalta. *IEEE Geoscience and Remote Sensing Letters*, 8(3):396–400, 2011.

[6] Pau Prats-Iraola, Rolf Scheiber, Marc Rodriguez-Cassola, Josef Mittermayer, Steffen Wollstadt, Francesco De Zan, Benjamin Bräutigam, Marco Schwerdt, Andreas Reigber, and Alberto Moreira. *IEEE Transactions on Geoscience and Remote Sensing*, 52(10):6003–6016, 2014.

[7] Giorgio Franceschetti and Riccardo Lanari. *Synthetic Aperture Radar Processing*. Electronic Engineering Systems Series. CRC Press, Boca Raton, FL, 1999.

[8] Ian G. Cumming and Frank H. Wong. *Digital Processing of Synthetic Aperture Radar Data. Algorithms and Implementation*. Artech House, Boston, 2005.