DIFFERENCE POTENTIAL METHODS FOR HYPERBOLIC PROBLEMS USING HIGH ORDER FINITE DIFFERENCE SCHEMES[1]

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We investigate the solution of hyperbolic initial BVPs by the method of difference potentials (DP) with the goal of achieving the same range of capabilities previously demonstrated for time-harmonic problems with general shape domains [1]. DP serves as a flexible and inexpensive avenue toward high order accuracy using finite differences (FD) on regular structured grids. It allows for non-conforming boundaries/interfaces with no loss of accuracy, and includes a general approach to boundary conditions. For hyperbolic problems, two approaches are presented. One may apply DP directly to a full formulation of the PDE (space + time). Alternatively, one may discretize in time first and then solve the resulting spatial equations by DP at each time step. Any high-order FD scheme in time/space may be used with either DP approach. We present a new compact FD scheme for the scalar wave equation which is 4th order in both time and space.

For the full formulation DP approach, the key difficulty is that the boundary extends as time elapses, making the computation of Calderon's projections increasingly expensive. However, by introducing an auxiliary problem that satisfies the (strong) Huygens' principle, we effectively truncate the temporal nonlocality of the problem to a finite non-increasing time interval. As a result, the overall computational complexity for long time simulations is even lower than that of a standard explicit scheme. In the case of separating the time and space discretizations, we use an implicit time scheme and solve the resulting spatial equations at each time step by DP. The computational cost for this method is comparable to a typical implicit FD scheme.

Both DP methods have been successfully implemented for the wave equation with Dirichlet BCs. Results show that high order accuracy in space and time are maintained even for nonconforming boundary shapes (e.g., a Cartesian grid with a spherical or circular boundary).

REFERENCES

[1] S. Britt, S. Tsynkov, E. Turkel. A high order numerical method for the Helmholtz equation with non-standard boundary conditions. *SIAM J. Sci. Comput.*, 35:A2255–A2292, 2013.

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