Detection of Material Dispersion Using SAR

Mikhail Gilman, Department of Mathematics, North Carolina State University, Box 8205, Raleigh, NC 27695, USA, phone: +1-919-515-3797, fax: +1-919-513-7336, mgilman@ncsu.edu
Semyon Tsynkov, Department of Mathematics, North Carolina State University, Box 8205, Raleigh, NC 27695, USA

Abstract

Measurement of material dispersion, i.e., the dependence of reflectivity upon frequency of electromagnetic wave, can provide useful information about the target material. Extension of the standard matched filtering approach to the response delay "coordinate" is complicated by the range-delay ambiguity of imaging and limited angular coherence of the target. We suggest to recover the above dependence in a simple parametrized form retaining the first two terms of the Taylor expansion of reflectivity in frequency. Corrections to the point spread function are computed, and possibilities of parameter extraction from image data are discussed.

1 Introduction

Standard synthetic aperture radar (SAR) provides information about target reflectivity as a function of two coordinates. Additional information about the target can be gained if for each point, dependence of reflectivity on frequency of electromagnetic wave, an effect also called material dispersion, is also recovered. This can provide an insight into material type and/or fine structure of the target [1, 2].

Considered in time domain, frequency dependence can be thought of as a delay of material response to the incident field, whereas an immediate response, i.e., a delta function in time, corresponds to a frequency-independent reflectivity. This delay can be treated as a third coordinate upon which the reflectivity depends. In fact, some publications considering recovery of 3D shapes from SAR (see, e.g., [3]) can be relevant in this context.

An approach to divide a SAR frequency band into several sub-bands and create an image for each sub-band, thus getting several points on the frequency axis corresponding to the central frequencies of each sub-band, has been tested in [4]. The framework of SAR ambiguity function [5, 6, 7] has been extended in [8, 9] to the case of frequency-dependent reflectivity, and expressions for point spread function (PSF) in coordinate-frequency domain have been obtained.

From implementation standpoint, wide angular apertures used in [4, 8, 9] require the correspondingly high angular coherence of the target, which may not be present, whereas narrow angular apertures exhibit range-delay ambiguities [2, 8]; these ambiguities also appear as wide sidelobes of a 3D PSF in [3]. Using ultra-wideband SAR in sub-band mode may also appear impractical from resolution and SNR considerations.

Our approach is to use a very simple parametrized form of the dependence of reflectivity on frequency, thus reducing the number of unknowns to be reconstructed for each coordinate pixel. This approach is suitable for the narrow-band and narrow-aperture (i.e., the most common) SAR. Similar technique can be used to retrieve the parameters of Faraday rotation from single-polarization SAR.

2 Imaging dispersive targets

2.1 Material Dispersion Parametrized

We assume that reflectivity of any material comprising the scene is represented by the first two terms of its Taylor expansion in frequency around the radar central frequency ω_0, i.e.,

$$s(\omega) = s_0 + s_1(\omega - \omega_0) \tag{1}$$

A non-dispersive material with $s_1 = 0$ is the one that has only immediate response to the incident field. Both s_0 and s_1 can be complex and can be obtained from the material and scattering models. A feasible material model can be a multiterm Debye model [1].

2.2 Dispersive PSF

Most common waveform used for high-resolution radars is a linear frequency-modulated signal, also called chirp, represented by

$$P(t) = A(t)e^{i\omega_0 t}; \quad A(t) = e^{i\alpha t^2} \text{ for } |t| < \frac{\tau}{2} \tag{2}$$

where α and τ are chirp rate and duration, respectively. The expression for instantaneous frequency is thus

$$\omega(t) = \omega_0 + 2\alpha t \tag{3}$$

so $|\omega - \omega_0| < B/2$, where $B = 2|\alpha|\tau$ is bandwidth.

When the signal reaches a target, its time dependence is given by a retarded potential expression:

$$\varphi(t) \sim \frac{P(t')}{R}; \quad t' = t - \frac{R}{c} \tag{4}$$

where R is the distance between the antenna and the target and c is the speed of light. In signal processing, the slow dependence on R in the denominator is usually neglected.

ISBN 978-3-8007-3607-2 / ISSN 2197-4403

In frequency domain, expression for the scattered field ψ due to a point scatterer with reflectivity $s(\omega)$ is

$$\psi(\omega) = \varphi(\omega) s(\omega) \tag{5}$$

For a large time-bandwidth product, i.e., for $|\alpha|\tau^2 \gg 1$, expression (5) can be approximated in time domain by

$$\psi(t) \sim P(t')(s_0 + 2\alpha s_1 t') \tag{6}$$

Expression (6) can be obtained either intuitively using the relation (3) in (5) or by taking inverse Fourier transform of (5).

From the scatterer, the signal propagates back to antenna according to the model (4). Matching the signal received by antenna to the emitted signal results in an image:

$$\begin{aligned} I(w) &\sim \int \overline{P(t-w)} P(t'')(s_0 + 2\alpha s_1 t'')dt \\ &\approx e^{i\omega_0(w-\eta)} \Big(s_0 \int_{-\tau/2}^{\tau/2} e^{4i\alpha u T} du \\ &\quad + 2\alpha s_1 \int_{-\tau/2}^{\tau/2} e^{4i\alpha u T} u du \Big) \end{aligned} \tag{7}$$

where the overbar denotes complex conjugate, $\eta = 2R/c$ is the signal round-trip time between the target and antenna, and

$$t'' = t - \eta; \quad T = \frac{w-\eta}{2}; \quad u = t - \frac{w+\eta}{2}$$

Straightforward calculation of (7) results in

$$I(w) \sim e^{i\omega_0(w-\eta)} \tau W(\xi) \tag{8}$$

where

$$W(\xi) = s_0 \operatorname{sinc}\xi + \frac{Bs_1}{2i} \operatorname{sinc}'\xi \tag{9}$$

and

$$\xi = BT; \quad \operatorname{sinc}\xi = \frac{\sin\xi}{\xi}$$

The second term in (9) is a contribution due to material dispersion; note that it is proportional to the derivative of the first (regular) term. Indeed, it can be seen that the last integral in (7) is proportional to the derivative of the preceding integral with respect to T.

2.3 Single-Pulse Image in Presence of Dispersion

For distributed targets, we assume a (continuous) set of locations η (represented by the corresponding signal round-trip time) such that s_0 and s_1 become functions of η. The imaging formula (8) becomes

$$\begin{aligned} I(w) &\sim \int e^{i\omega_0(w-\eta)} \operatorname{sinc} \frac{B(w-\eta)}{2} s_0(\eta)\, d\eta \\ &+ \int e^{i\omega_0(w-\eta)} \operatorname{sinc}' \frac{B(w-\eta)}{2} \frac{Bs_1(\eta)}{2i}\, d\eta \end{aligned} \tag{10}$$

In expression (10), we can recognize two windowed Fourier transforms (WFT) of functions s_0 and s_1 with sinc and sinc' windows, respectively, centered at w and evaluated at the Bragg spatial frequency [the latter can be seen from definition of η after (7)]. However, as long as $\omega_0 \gg B$, the scale of the first term in the integrands in (10) is much smaller than that of the second. We can consider segments of intermediate scales V such that $\omega_0^{-1} \ll V \ll B^{-1}$, so the arguments of sinc and sinc' may be assumed constant and pulled out of the integrals; what remains evaluates to Fourier transform of s_0 and s_1 limited to this segment. Such a coordinate-frequency representation reduces to

$$\begin{aligned} e^{-i\omega_0 w} I(w) &\sim \int \operatorname{sinc} \frac{B(w-\eta)}{2} s_0^{\mathrm{B}}(\eta)\, d\eta \\ &+ \int \operatorname{sinc}' \frac{B(w-\eta)}{2} \frac{Bs_1^{\mathrm{B}}(\eta)}{2i}\, d\eta \end{aligned} \tag{11}$$

where

$$s_{0,1}^{\mathrm{B}}(\eta) = \frac{1}{V} \int_{\eta-V/2}^{\eta+V/2} e^{-i\omega_0\eta'} s_{0,1}(\eta')\, d\eta' \tag{12}$$

is a WFT of $s_{0,1}(\eta)$ evaluated at the Bragg spatial frequency with a rectangular window. Other types of windows can be adapted in (12); however, the sensitivity of the procedure to the shape and side of windows has yet to be explored.

2.4 Imaging in Azimuth

The procedure of SAR imaging assumes that single-pulse images (7) are taken for multiple locations of the antenna track and then processed coherently in slow time to achieve azimuthal resolution (see, e.g., [6] and references therein). For broadside imaging, the distance R between the antenna and the target varies slowly from one pulse to another, and it was shown [10] that accurate to a B/ω_0 factor, imaging in range retains its convolution form (11) with some redefinition of target reflectivity functions $s_{0,1}^{\mathrm{B}}$. Namely, the new values of $s_{0,1}^{\mathrm{B}}$ are obtained by convolution of original values with the azimuthal ambiguity function, and equation (11) is considered for each azimuth line.

3 Comparison to the Faraday Rotation Case

The effect of a significant Faraday rotation on SAR imaging [10] has some similarity with that of material dispersion. Faraday rotation is a rotation of polarization plane for an electromagnetic wave propagating in anisotropic medium (e.g., ionospheric plasma). For an antenna operating on a single linear polarization, the effective received antenna signal ψ_F in the presence of the rotation is related to the non-rotated signal ψ as

$$\psi_F(t') = \psi(t') \cos\varphi_F(\omega), \tag{13}$$

where φ_F is the rotation angle. For the two-way propagation in a magnetized plasma, the angle φ_F depends on

ISBN 978-3-8007-3607-2 / ISSN 2197-4403

the instantaneous frequency as [11]

$$\varphi_F(\omega) = 2 \cdot \frac{R}{2c} \frac{\omega_{\rm pe}^2 \Omega \cos\beta}{\omega^2} \propto \omega^{-2} \tag{14}$$

where $\omega_{\rm pe}$ is the Langmuir frequency, Ω is the electron cyclotron frequency, and β is the angle between the ray path and the magnetic field. Using (14), the $\cos\varphi_F(\omega)$ factor in (13) can be expanded around ω_0 similarly to (1). Instead of (6) we get

$$\psi_F(t) \sim P(t')\Big(p + \frac{2q}{\tau}t'\Big)s_0 \tag{15}$$

where

$$p = \cos\varphi_F(\omega_0); \quad q = \frac{B}{\omega_0}\varphi_F(\omega_0)\sin\varphi_F(\omega_0)$$

and we have assumed that the target is non-dispersive. The imaging formula, instead of (11), will be

$$\begin{aligned} e^{-i\omega_0 w} I(w) \sim \int\Big(&\operatorname{sinc}\frac{B(w-\eta)}{2} \\ &-Q\operatorname{sinc}'\frac{B(w-\eta)}{2}\Big)s_0^{\rm B}(\eta)\,d\eta \end{aligned} \tag{16}$$

where $Q = q/p$. The sinc' term in (16) is due to the Faraday rotation and vanishes for isotropic medium where $\varphi_F = 0$. The effect of sinc' terms in (16) and in (11) is broadening of the main lobe of the imaging kernel.

The difference between the cases of material dispersion and the Faraday rotation can be seen in that for the linear in t' function in the round brackets in (15), the coefficients are the same for all target locations, whereas for the linear function in (6), these coefficients are functions of η. Hence, a single corrected filter can be built that will match the signal in the Faraday rotation case [10], while for the case of material dispersion, a filter matching the signal scattered from one part of the image will have a mismatch in other parts of the image.

4 Detection and Evaluation of Target Dispersion

There does not seem to exist an obvious procedure of extracting two unknown functions $s_0^{\rm B}(\eta)$ and $s_1^{\rm B}(\eta)$ from a single equation (11). However, we can formulate certain assumptions where some steps towards detection of dispersion and evaluation of its parameters can be made.

4.1 Dispersive Point Targets

Suppose there is an extremely bright and very small target such that reflectivity of all other targets in some neighborhood is negligibly small. A model for such a target will be

$$\begin{pmatrix} s_0^{\rm B}(\eta) \\ s_1^{\rm B}(\eta)\end{pmatrix} = \begin{pmatrix} S_0 \\ S_1 \end{pmatrix}\delta(\eta-\eta_0)$$

This model has three unknown constants: S_0, S_1, and η_0. The relation (11) becomes

$$\begin{aligned} e^{-i\omega_0 w} I(w) \sim \Big(&\operatorname{sinc}\frac{B(w-\eta_0)}{2}S_0 \\ &+\operatorname{sinc}'\frac{B(w-\eta_0)}{2}\frac{BS_1}{2i}\Big) \end{aligned} \tag{17}$$

Given this relation, we can sample $I(w)$ at many values of w to build a system of equations that can be solved in a weak sense for η_0, S_0, and S_1. Note that the $e^{-i\omega_0 w}$ term in this relation changes rapidly, so the system is better conditioned if only the absolute values of its left- and right-hand sides are used.

An alternative "brute force" method would be to try varying the matched filter. We can consider a family of filters $\{P_Z\}$ parametrized by a complex-valued Z in the form

$$P_Z(t) = P(t)\Big(1 + \frac{Z}{\tau}t\Big)$$

and use $\overline{P_Z}$ instead of $\overline{P}$ to build an image. If a point target is imaged sharpest for some $Z = Z_0 \neq 0$, then we can argue that this filter matches the returned signal, so

$$\frac{2\alpha S_1}{S_0} = \frac{\overline{Z_0}}{\tau}$$

see (7). So, the ratio of the two parameters involved, i.e., S_1/S_0, is reconstructed.

4.2 Interface Between Non-Dispersive and Dispersive Material

Areas with constant properties delineated by sharp interfaces could be anticipated for anthropogenic targets. Similarly to a point target case, a sharp interface allows to explore the properties of the imaging kernel.

Let an interface be at $\eta = \eta_0$ such that the parameters of reflectivity function have the form:

$$\begin{aligned} s_0^{\rm B}(\eta) &= S_- + (S_+ - S_-)\theta(\eta-\eta_0) \\ s_1^{\rm B}(\eta) &= S_1\theta(\eta-\eta_0) \end{aligned} \tag{18}$$

where $\theta(\eta)$ is a step function, and S_-, S_+, and S_1 are constants. Integration of (11) with (18) yields

$$\begin{aligned} &e^{-i\omega_0 w} I(w) \sim \\ &\quad S_- + (S_+ - S_-)\Big[\frac{1}{\pi}\operatorname{Si}\Big(B\frac{w-\eta_0}{2}\Big) + \frac{1}{2}\Big] \\ &\quad - i\frac{BS_1}{2\pi}\operatorname{sinc}B\frac{w-\eta_0}{2} \end{aligned} \tag{19}$$

where Si is a sine integral function:

$$\operatorname{Si}(\xi) = \int_0^\xi \operatorname{sinc}(\xi')\,d\xi'$$

As in the case of dispersive point targets (Section 4.1), relation (19) can be sampled at several values of w, and the resulting system can be solved in a weak sense for η_0, S_-, S_+, and S_1.

ISBN 978-3-8007-3607-2 / ISSN 2197-4403

4.3 Effect of Weighting Windows

Smoothing windows are added to the matched filters to reduce the PSF sidelobes [12, Section 2.6]. The window function $H(t)$ appears in (7) as an additional factor in the integrands:

$$\begin{aligned} I(w) &\sim \int \overline{P(t-w)} H(t-w) P(t'')(s_0 + 2\alpha s_1 t'') dt \\ &\approx e^{i\omega_0 (w-\eta)} \Big(s_0 \int_{-\tau/2}^{\tau/2} e^{4i\alpha u T} H(u)\, du \\ &\qquad + 2\alpha s_1 \int_{-\tau/2}^{\tau/2} e^{4i\alpha u T} H(u)\, u du \Big) \end{aligned} \tag{20}$$

Typically, $H(t)$ is a real-valued even function that is equal one at $t = 0$ and monotonically decreases for $0 < t < \tau/2$; this function replaces the rectangular window $\theta(|t - \tau/2|)$ in (7). Correspondingly, the sinc and sinc' functions in (9) and the subsequent formulas will be replaced by the Fourier transform $\hat{H}(\xi)$ of the window function and its derivative $\hat{H}'(\xi)$, respectively. Usage of sidelobe-suppressing windows for building an image and in (17) and (19) should have an advantage over the rectangular window in (7) and the resulting sinc-based kernel of (11) because of lower image artifacts, especially in the vicinity of points where the reflectivity function is discontinuous, as in the cases considered in Sections 4.1 and 4.2.

5 Conclusions

The idea of matched filtering is based on a certain model of scatterer. The standard matched filter $\overline{P}(\ldots)$, see (7), is built on an assumption that the scattering is instantaneous. If this is not the case, i.e., the material is dispersive, then this filter does not match the returned signal, which results in distortions of the image. The procedure of recovering two unknown reflectivity functions $s_0(\eta)$ and $s_1(\eta)$ from a single equation (11) is unlikely to be found. Hence, we should either increase the number of equations (i.e., build multiple images) by a variety of imaging filters or use simplifying assumptions about the unknown functions.

6 Acknowledgements

This work was supported by the US Air Force Office of Scientific Research (AFOSR) under agreement FA9550-10-1-0092.

References

[1] Richard Medina, John Penn, and Richard Albanese: *Dielectric response data on materials of military consequence*, Technical Report AFRL-HE-BR-TR-2002-0155, United States Air Force Research Laboratory, 2002.

[2] Victor C. Chen and Hao Ling: *Time-frequency transforms for radar imaging and signal analysis*, Artech House, 2002.

[3] Linda J. Moore and Lee C. Potter: *Three-dimensional resolution for circular synthetic aperture radar*, International Society for Optics and Photonics, Defense and Security Symposium, p. 656804, 2007.

[4] Richard A. Albanese and Richard L. Medina: *Materials identification synthetic aperture radar: progress toward a realized capability*, Inverse Problems, Vol. 29:054001, 2013.

[5] L. J. Cutrona: *Synthetic aperture radar*, in M. Skolnik, editor, *Radar Handbook*, McGraw-Hill, 1990.

[6] Margaret Cheney: *A mathematical tutorial on synthetic aperture radar*, SIAM Review, Vol. 43, pp.301–312, 2001.

[7] Mikhail Gilman, Erick Smith, and Semyon Tsynkov: *Reduction of ionospheric distortions for spaceborne SAR with the help of image registration*, Inverse Problems, Vol. 29:054005, 2013.

[8] Margaret Cheney: *Imaging frequency-dependent reflectivity from synthetic-aperture radar*, Inverse Problems, Vol. 29:054002, 2013.

[9] Paul Sotirelis, Jason Parker, Xueyu Hu, Margaret Cheney, and Matthew Ferrara: *Frequency-dependent reflectivity image reconstruction*, Proc. SPIE 8746, Algorithms for Synthetic Aperture Radar Imagery XX, p. 874602, 2013

[10] Mikhail Gilman, Erick Smith, and Semyon Tsynkov: *Single-polarization SAR imaging in the presence of Faraday rotation*, Inverse problems, submitted for publication.

[11] V. L. Ginzburg: *The Propagation of Electromagnetic Waves in Plasmas*, Pergamon Press, 1964.

[12] Ian G. Cumming and Frank H. Wong: *Digital Processing of Synthetic Aperture Radar Data*, Artech House, 2005.

ISBN 978-3-8007-3607-2 / ISSN 2197-4403