Cross-Channel Contamination of PolSAR Images due to Frequency Dependence of Faraday Rotation Angle

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***Abstract*—Compensation of the Faraday rotation (FR) effect in polarimetric synthetic aperture radar (PolSAR) involves a rotation matrix, with the FR angle determined by the magnetic field, total electron content, signal frequency, and propagation direction. We analyze the conditions where the signal frequency and/or propagation direction cannot be considered constants. In other words, the rotation matrix based on the main look direction and central radar frequency may have a significant mismatch with the received signal in fast or slow time. We derive estimates for the resulting polarimetric distortions and their effect on applications such as instrument calibration in space and measurement of the aboveground biomass.**

***Index Terms*—polarimetric synthetic aperture radar, Faraday effect, matched filters, biomass**

I. Introduction

Usage of SAR for applications such as ground and foliage penetration, biomass measurement, etc., mandates usage of low carrier frequencies, possibly in UHF/VHF range, where the effect of ionosphere on imaging cannot be ignored [3], [9], [12]. Ionospheric plasma is dispersive and gyrotropic, and propagation of radar pulses through such a medium results in their modification, such that the signal received by the antenna does not match the emitted signal. Modifications to the signal processing procedure are required to preserve the system characteristics such as resolution. A number of publications has been devoted to the mitigation of the effect of dispersive propagation through the ionospheric plasma, see, e.g., [2], [6], [16] and references therein. As far as the Faraday rotation (FR) is concerned, the discussion is mostly concentrated around estimating the rotation angle. Given the rotation angle, the standard correction procedure is application of the rotation matrix to the vector of received polarimetric channels, see. e.g., [13], [14].

In this work, we consider the case where dispersive effects are corrected and focus on FR, specifically, on the dependence of the rotation angle on propagation direction and instantaneous frequency. In order to increase resolution in the range and azimuthal direction, the bandwidth and, respectively, aperture length should be increased. Each of these increases potentially leads to an increase of the range of rotation angles for a single image acquisition. If, for example, the emitted signal is linear frequency modulated (LFM) where the instantaneous frequency is increasing (the so-called "upchirp"), then the low-frequency part emitted early during the transmit period will be rotated by a bigger angle than the high-frequency part emitted later. Hence, the contribution of one scattering channel into the received signal in another channel becomes frequency-dependent. Due to a one-to-one relation between the signal time (also called fast time) and instantaneous frequency, this dependence can equivalently be formulated as a dependence of the FR angle on fast time. Similarly, movement of the antenna over the aperture leads to variation of the propagation angle w.r.t. the direction of the external magnetic field; this may be treated as a dependence of the FR angle on slow time. Neither of these situations can adequately be described by a single rotation matrix. Because of this, the inversion using a single rotation matrix will be inaccurate and, hence, will result in the filter mismatches and, eventually, distortions in the reconstruction of polarimetric properties of the target.

In [7], we have considered a procedure that takes into account the dependence of the FR angle on fast and slow times. A new signal processing kernel, henceforth called the polarimetric matched filter (PMF), has been suggested. The PMF combines the standard matched phase multiplier with two rotation matrices, in which the rotation angle depends on slow and fast times. We have shown that usage of PMF instead of the traditional procedure significantly improves polarimetric fidelity of the resulting images. Here we analyze the effect of the varying FR angle on the system calibration in space and on the biomass measurement. In particular, we demonstrate that for a certain combination of system and ionospheric parameters, a popular formula for the assessment of aboveground forest biomass from the backscattering intensity in the HV channel can be seriously compromised when used with the traditional

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signal processing. Hence, transition to the PMF processing will be justified because it almost completely eliminates this kind of distortions.

II. Propagation of radar pulses in the presence of the Faraday rotation

Scattering by a point target located at $\boldsymbol{z}$ is described by the matrix $\mathbf{S}$ that relates the horizontal and vertical components of the incident field $\boldsymbol{E}^{\mathrm{i}}$ and scattered field $\boldsymbol{E}^{\mathrm{s}}$:

$$\begin{pmatrix} E^{\mathrm{s}}_{\mathrm{H}} \\ E^{\mathrm{s}}_{\mathrm{V}} \end{pmatrix}(t,\boldsymbol{z}) = \begin{pmatrix} S_{\mathrm{HH}} & S_{\mathrm{HV}} \\ S_{\mathrm{VH}} & S_{\mathrm{VV}} \end{pmatrix} \cdot \begin{pmatrix} E^{\mathrm{i}}_{\mathrm{H}} \\ E^{\mathrm{i}}_{\mathrm{V}} \end{pmatrix}(t,\boldsymbol{z}), \qquad (1)$$

or $\boldsymbol{E}^{\mathrm{s}}(t,\boldsymbol{z}) = \mathbf{S}\boldsymbol{E}^{\mathrm{i}}(t,\boldsymbol{z})$. For distributed targets in a single-scattering (or Born) approximation, entries of this matrix can be functions of the target coordinate, i.e., $\mathbf{S} = \mathbf{S}(\boldsymbol{z})$.

In turn, propagation of the signal emitted from the antenna position $\boldsymbol{x}$ to the target at $\boldsymbol{z}$ through gyrotropic medium is characterized by propagation delay and rotation of the polarization plane:

$$\boldsymbol{E}^{\mathrm{i}}(t,\boldsymbol{z}) = K\mathbf{R}(\varphi_{\mathrm{F}}) \cdot \boldsymbol{E}^{\mathrm{i}}(t - R_{\boldsymbol{z}}/c, \boldsymbol{x}), \qquad (2)$$

where we have ignored the difference between the speed of light and the phase speed of propagating waves because we assume that the dispersion-related effects are compensated (see [6, Chapter 3] for more detail). In (2), $R_{\boldsymbol{z}} = |\boldsymbol{z} - \boldsymbol{x}|$; K is the attenuation factor, and

$$\mathbf{R}(\varphi_{\mathrm{F}}) \stackrel{\mathrm{def}}{=} \begin{pmatrix} \cos\varphi_{\mathrm{F}} & \sin\varphi_{\mathrm{F}} \\ -\sin\varphi_{\mathrm{F}} & \cos\varphi_{\mathrm{F}} \end{pmatrix}. \qquad (3)$$

In (3), the Faraday rotation angle φ_{F} for propagation through magnetized plasma is defined as

$$\varphi_{\mathrm{F}} = -\frac{R_{\boldsymbol{z}}}{2c}\frac{\omega_{\mathrm{pe}}^2 \Omega_{\mathrm{e}} \cos\beta}{\omega^2}, \qquad (4)$$

where the electron plasma frequency and gyrofrequency are given by

$$\omega_{\mathrm{pe}}^2 = \frac{4\pi N_{\mathrm{e}} e^2}{m_{\mathrm{e}}} \quad \text{and} \quad \Omega_{\mathrm{e}} = -\frac{e|\boldsymbol{H}_0|}{m_{\mathrm{e}} c}, \qquad (5)$$

respectively, N_{e} is the electron number density, $-e$ and m_{e} are the electron charge and mass, respectively, and β is the angle between the propagation direction and the external magnetic field $\boldsymbol{H}_0$. For an inhomogeneous medium such as the Earth's ionosphere, we assume that the electron number density and external magnetic field are averaged over the propagation path.

We specify the incident waveform in either polarization as a linear frequency modulated (LFM, or chirp) signal

$$\boldsymbol{E}(t,\boldsymbol{x}) = \boldsymbol{E}_{(\mathrm{H,V})} P(t), \text{ where } P(t) = A(t) e^{-i\omega_0 t}, \qquad (6)$$

$E_{(\mathrm{H})} = (E_0, 0)^{\mathrm{T}}$ and $E_{(\mathrm{V})} = (0, E_0)^{\mathrm{T}}$ are the linear polarization vectors, and

$$A(t) = \chi_\tau(t) e^{-i\alpha t^2}, \quad \chi_\tau(t) = \begin{cases} 1, & t \in [-\tau/2, \tau/2], \\ 0, & \text{otherwise.} \end{cases} \qquad (7)$$

For instantaneous frequency of the signal, we have

$$\omega(t) = \omega_0 + 2\alpha t = \omega_0 + \frac{B}{\tau}t, \quad |t| \leqslant \frac{\tau}{2}, \qquad (8)$$

where B is the bandwidth: $|\omega(t) - \omega_0| \leqslant B/2$.

We will calculate two reflected field vectors $\boldsymbol{E}^{\mathrm{s}}$ given by (1) resulting from the field emitted in two basic linear polarizations. Then we combine these vectors into a single data matrix $\mathbf{M}$ and will take its values at the antenna position $\boldsymbol{x}$. In two-way signal propagation, the travel delay and Faraday rotation are applied to the signal twice. Denote the retarded time by

$$\mathfrak{t} = t - 2R_{\boldsymbol{z}}/c.$$

Then, ignoring the difference between the speed of propagation of the pulse envelope (i.e., the group speed, see [6, Chapter 5] for more detail) and the speed of light, we have

$$\mathbf{M}(t,\boldsymbol{x}) = \int P(\mathfrak{t})\mathbf{R}(\varphi_{\mathrm{F}}) \cdot \mathbf{S}(\boldsymbol{z}) \cdot \mathbf{R}(\varphi_{\mathrm{F}})\, d\boldsymbol{z}. \qquad (9)$$

In this equation, we have incorporated the constant E_0 and the slowly varying attenuation factor K into the matrix $\mathbf{S}$. The FR angle in (9) depends on several variables:

$$\varphi_{\mathrm{F}} \equiv \varphi_{\mathrm{F}}(t,\boldsymbol{x},\boldsymbol{z}) = -\frac{|\boldsymbol{x} - \boldsymbol{z}|}{2c}\frac{\omega_{\mathrm{pe}}^2 \Omega_{\mathrm{e}} \cos\beta(\boldsymbol{x},\boldsymbol{z})}{\omega^2(\mathfrak{t}(t,\boldsymbol{x},\boldsymbol{z}))}, \qquad (10)$$

see (4), where $\omega(\mathfrak{t})$ is defined by (8). We will consider variations of $\boldsymbol{z}$ of the order of resolution, while $\boldsymbol{x}$ varies at the scale of the synthetic aperture; hence, we will ignore the dependence of φ_{F} on $\boldsymbol{z}$.

The task of the polarimetry is to reconstruct the matrix $\mathbf{S}(\boldsymbol{z})$ from the data $\mathbf{M}(t,\boldsymbol{x})$.

III. Two approaches to signal processing

In traditional polarimetry (see, e.g., [11]), images in all channels are built independently. Mathematically, it can be expressed as

$$Y_{pq}(\boldsymbol{y}) = \iint \overline{P(\mathfrak{t})} M_{pq}(t,s)\, dt\, ds, \qquad (11)$$

where $pq \in \{\mathrm{HH}, \mathrm{HV}, \mathrm{VH}, \mathrm{VV}\}$, the slow time s parametrizes the antenna trajectory: $M(t,s) \equiv M(t,\boldsymbol{x}(s))$, and $\mathfrak{t} \equiv \mathfrak{t}(t,s,\boldsymbol{y}) = t - 2|\boldsymbol{x}(s) - \boldsymbol{y}|/c$. Here, $\overline{P(\mathfrak{t})}$ is the standard matched filter that can also be used in scalar imaging. Further, if the ionospheric conditions are found such that the FR is present, then the matrix resulting from (11) is "rotated" as follows:

$$\mathbf{I}^{\mathrm{trad}}(\boldsymbol{y}) = \mathbf{R}(-\varphi_{\mathrm{F}}^*) \cdot \mathbf{Y}(\boldsymbol{y}) \cdot \mathbf{R}(-\varphi_{\mathrm{F}}^*). \qquad (12)$$

The constant rotation angle φ_{F}^* above can be calculated using $\omega = \omega_0$ and the center of the synthetic aperture as $\boldsymbol{x}$ in (4).

By contrast, in what we call application of a polarimetric matched filter (PMF) [7], we perform rotation and matched filtering in a single step by taking into account the dependence of the rotation angle on the fast and slow times:

$$\mathbf{I}^{\mathrm{PMF}}(\boldsymbol{y}) = \int \overline{P(\mathfrak{t})}\mathbf{R}(-\varphi_{\mathrm{F}}) \cdot \mathbf{M}(t,s) \cdot \mathbf{R}(-\varphi_{\mathrm{F}})\, dt\, ds, \qquad (13)$$

where $\varphi_{\mathrm{F}} \equiv \varphi_{\mathrm{F}}(t,s,\boldsymbol{y})$ is obtained with the help of (10) by using $\boldsymbol{x}(s)$ and $\boldsymbol{y}$ in place of $\boldsymbol{x}$ and $\boldsymbol{z}$, respectively.

IV. Imaging operator

The imaging operator $\mathbf{S}(\boldsymbol{z}) \mapsto \mathbf{I}(\boldsymbol{y})$ for both traditional and PMF approached can be represented as

$$I_{ij}(\boldsymbol{y}) = \sum_{kl} \int W_{iklj}(\boldsymbol{y},\boldsymbol{z}) S_{kl}(\boldsymbol{z})\, d\boldsymbol{z}, \tag{14}$$

where $i,j,k,l \in \{\mathrm{H},\mathrm{V}\}$. The kernel of the imaging operator in (14) can be calculated by taking into account expression (9) for the reflected signal:

$$\begin{aligned} W_{iklj}(\boldsymbol{y},\boldsymbol{z}) = \iint & \overline{A(\mathsf{t}(t,s,\boldsymbol{y}))} A(\mathsf{t}(t,s,\boldsymbol{z})) \\ & e^{i\Phi} R_{ik}(\Delta\varphi_{\mathrm{F}}) R_{lj}(\Delta\varphi_{\mathrm{F}}) dt\, ds, \end{aligned} \tag{15}$$

where $\Phi = -2\omega_0(|\boldsymbol{x}(s)-\boldsymbol{y}| - |\boldsymbol{x}(s)-\boldsymbol{z}|)/c$, and we have two different expression for $\Delta\varphi_{\mathrm{F}}$:

$$\Delta\varphi_{\mathrm{F}}^{\mathrm{trad}} = \varphi_{\mathrm{F}}(t,s,\boldsymbol{z}) - \varphi_{\mathrm{F}}^*$$

and

$$\Delta\varphi_{\mathrm{F}}^{\mathrm{PMF}} = \varphi_{\mathrm{F}}(t,s,\boldsymbol{z}) - \varphi_{\mathrm{F}}(t,s,\boldsymbol{y}).$$

Ideally, the matrix structure of W should follow

$$W_{iklj} \propto \delta_{ik}\delta_{lj}, \tag{16}$$

whereas any other non-zero term of the tensor W describes a contamination of one polarimetric channel by another channel. The latter amounts to a distortion of the polarimetric reconstruction.

V. Analysis of distortions

In order to perform the analysis, we assume a linear flight trajectory at constant height H: $\boldsymbol{x}(s) = (sL_{\mathrm{SA}}, -L, H)$, $|s| \leqslant 1/2$, where L_{SA} is the length of the synthetic aperture, L is the distance from the target to the ground track, and the origin of coordinates is in the target area. We assume that $L_{\mathrm{SA}} \ll R$, where $R = (L^2+H^2)^{1/2}$.

Due to the form of the rotation matrix $\mathbf{R}$ given by (3), the tensor W has three essentially different entries:

$$\begin{aligned} W_{0,1,2}(\boldsymbol{y},\boldsymbol{z}) = \iint & \overline{A(\mathsf{t}(t,s,\boldsymbol{y}))} A(\mathsf{t}(t,s,\boldsymbol{z})) \\ & e^{i\Phi} f_{0,1,2}(\Delta\varphi_{\mathrm{F}}) dt\, ds, \end{aligned} \tag{17}$$

where

$$\begin{aligned} f_0(\Delta\varphi_{\mathrm{F}}) &= \cos^2\Delta\varphi_{\mathrm{F}}, \\ f_1(\Delta\varphi_{\mathrm{F}}) &= \cos\Delta\varphi_{\mathrm{F}} \sin\Delta\varphi_{\mathrm{F}}, \\ f_2(\Delta\varphi_{\mathrm{F}}) &= \sin^2\Delta\varphi_{\mathrm{F}}. \end{aligned} \tag{18}$$

The "non-contaminating" (in the sense of (16)) entries of the imaging kernel are given by $W_{iijj}(\boldsymbol{y},\boldsymbol{z}) = W_0(\boldsymbol{y},\boldsymbol{z})$ for any i and j, while the "contaminating" entries are given by the other two functions, e.g., $W_{\mathrm{HHHV}}(\boldsymbol{y},\boldsymbol{z}) = W_1(\boldsymbol{y},\boldsymbol{z})$, $W_{\mathrm{HVVH}}(\boldsymbol{y},\boldsymbol{z}) = -W_2(\boldsymbol{y},\boldsymbol{z})$, etc. Hence, the level of distortions may be characterized using the norms of W_j, e.g., $\|W_1\|^2/\|W_0\|^2$ and $\|W_2\|^2/\|W_0\|^2$, where $\|\cdot\|$ is the L_2-norm: for $f(\boldsymbol{y},\boldsymbol{z}) = f(\boldsymbol{y}-\boldsymbol{z})$ (which will be the case for all entries of W with both the traditional and PMF type processing), it is defined as

$$\|f\|^2 = \iint_{(\boldsymbol{z},\mathbf{e}_3)=0} |f(\boldsymbol{z})|^2\, d\boldsymbol{z}. \tag{19}$$

The expression for the dominant term of $\Delta\varphi_{\mathrm{F}}$ in the case of traditional processing is different from the case of PMF. For the former, we have

$$\Delta\varphi_{\mathrm{F}}^{\mathrm{trad}} = \eta_{\mathrm{A}} s + \eta_{\mathrm{R}} t/\tau, \tag{20}$$

where

$$\eta_{\mathrm{A}} = -\varphi_{\mathrm{F}_0}(\mathbf{e}_{\boldsymbol{H}},\mathbf{e}_1)\frac{L_{\mathrm{SA}}}{R}, \quad \eta_{\mathrm{R}} = -\varphi_{\mathrm{F}_0}\frac{2B}{\omega_0}\cos\beta^*,$$

$\mathbf{e}_{\boldsymbol{H}} = \boldsymbol{H}_0/|\boldsymbol{H}_0|$, β^* is the angle between $\mathbf{e}_{\boldsymbol{H}}$ and the baseline propagation direction vector $-\boldsymbol{x}(0)$, and

$$\varphi_{\mathrm{F}_0} = -\frac{R}{2c}\frac{\omega_{\mathrm{pe}}^2\Omega_{\mathrm{e}}}{\omega_0^2}.$$

Formula (17) transforms into

$$\begin{aligned} W_j(\boldsymbol{y},\boldsymbol{z}) = \tau e^{i\Phi} & \int_{-1/2}^{1/2} ds \int_{-\tau/2}^{\tau/2} dt \\ & e^{-2i\xi_{\mathrm{A}}s - 2i\xi_{\mathrm{R}}t/\tau} f_j(\eta_{\mathrm{A}}s + \eta_{\mathrm{R}}t/\tau), \end{aligned} \tag{21}$$

where

$$\xi_{\mathrm{A}} = \frac{\omega_0(y_1-z_1)L_{\mathrm{SA}}}{Rc}, \quad \xi_{\mathrm{R}} = \frac{B(y_2-z_2)L}{Rc}, \tag{22}$$

and $j \in \{0,1,2\}$. Using (18), functions W_j can be computed either numerically or analytically [7].

We will consider here a hypothetical high-resolution P-band SAR system with the following parameters: $\omega_0/(2\pi) = 300\mathit{MHz}$, $B/(2\pi) = 8\mathit{MHz}$, $\tau = 5\cdot 10^{-5}s$, $H = 500\mathit{km}$, $L_{\mathrm{SA}} = 50\mathit{km}$, and $\theta = 60°$. As compared to the planned BIOMASS mission, see [10], it has the carrier frequency 1.45 times smaller and the bandwidth 1.33 times larger. Taking a high value of the total electron content (TEC) of $50\mathit{TECU} \equiv 5\cdot 10^{13} cm^{-2}$, and $\beta^* = 0$, we obtain $|\eta_{\mathrm{R}}| \approx 0.7$, $\eta_{\mathrm{A}} = 0$. The level of distortions can now be assessed by substituting the numerical values of η_{R} and η_{A} into (21). Although the distortions are characterized by the parameter η_{R} that is not small for our choice of parameters, their impact depends on the application.

Consider first the problem of radiometric calibration of the instrument while in space. A typical external calibration procedure includes corner reflectors on the ground (see [18, Section 4.4], and also [4], [8]). Since corner reflectors don't reflect cross-polarizations, we are interested only in the values of W_{HVVH} and W_{VHHV} of the tensor W in (14)–(15). Both these entries are expressed via f_2 in (18). Calculating the norms of (21) as shown in (19), we obtain $\|W_2\|^2/\|W_0\|^2 \approx 3\cdot 10^{-3}$; we can get for this ratio even a smaller value of $\approx 1.4\cdot 10^{-3}$ if we consider the energy in a single pixel and, accordingly, restrict integration in (19) further to the domain of $|\xi_{\mathrm{A}}| \leqslant \pi$ and $|\xi_{\mathrm{R}}| \leqslant \pi$, see (22). Although these levels of distortions are small by themselves, they are still comparable to the maximum level of cross-talk between the polarimetric channels selected for the future BIOMASS mission [15].

The situation gets more aggravated if a strongly reflecting channel contaminates a weak channel. For example, a popular scaling formula for the aboveground biomass uses the backscattering cross-section in the HV channel as an input, see [15]. The signal in this channel will be influenced by the co-polarized channels with much higher reflectivity. Accordingly, the relation between the contaminated and true measurements of the biomass b may be expressed as

$$\frac{b_{\text{cont}}}{b_{\text{true}}} \sim \left(\frac{|S_{\text{HV}}|^2 + Q(|S_{\text{HH}}|^2 + |S_{\text{VV}}|^2)}{|S_{\text{HV}}|^2} \right)^p, \tag{23}$$

where $p \approx 2.4$ [15], and $Q = (1/2)\|W_1\|^2/\|W_0\|^2$, see (17). In (23), we will use $|S_{\text{HH}}|^2/|S_{\text{HV}}|^2 \sim 10$ for the forested areas, see [5, Table I], and $Q \approx 2 \cdot 10^{-2}$ according to formula (21). This results in $b_{\text{cont}}/b_{\text{true}} \sim 1.2^{2.4} \approx 1.5$, i.e., an error of 50%.

The analysis above can be an indication that for the biomass measurements involving high-resolution low-frequency PolSAR with traditional signal processing, careful control of ionospheric distortions is necessary. Note that the calibration of the SAR system using images of distributed targets such as Amazon rainforest, see, e.g., [1], [17], can only partially alleviate this problem because the ionospheric conditions are subject to changes. Alternatively, we can always get reliable polarimetric measurements if we process the signal using the PMF of (13). Indeed, for the PMF, the dominant term of $\Delta\varphi_{\text{F}}$ is

$$\Delta\varphi_{\text{F}}^{\text{PMF}} = (\boldsymbol{p}, \boldsymbol{z} - \boldsymbol{y}), \tag{24}$$

where

$$\boldsymbol{p} = \frac{\mathbf{e}_{\boldsymbol{H}}}{R} + 4\frac{B\cos\beta^*}{\omega_0 c\tau}\frac{\partial|\boldsymbol{z} - \boldsymbol{x}(0)|}{\partial \boldsymbol{z}}\Big|_{\boldsymbol{z}=\boldsymbol{0}} \tag{25}$$

(for the system described above, the first term in (25) is much smaller than the second). The analysis carried out in [7] leads to the following estimate:

$$\frac{\|W_1\|^2}{\|W_0\|^2} \lesssim \max\Big((\boldsymbol{p}, \mathbf{e}_1)^2 L_{\text{SA}}\Delta_{\text{A}}, (\boldsymbol{p}, \mathbf{e}_2)^2 c\tau\Delta_{\text{R}}\frac{R}{2L}\Big). \tag{26}$$

The terms on the right hand side of (26) are of the same order, and each is about three orders of magnitude smaller than that for the traditional processing. This level of distortions is negligibly small.

VI. Conclusion

In SAR imaging through the magnetized ionosphere, the Faraday rotation angle may depend on fast and slow time. The effect on PolSAR is equivalent to the channel cross-talk: the measurements in one polarimetric channels have contributions from other channels. Calculations demonstrate that in certain situations, this effect cannot be ignored. A mitigation procedure, called the polarimetric matched filter, is suggested; its usage lowers the level of distortions of polarimetric reconstruction by several orders of magnitude.

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