On SAR Imaging through the Earth's Ionosphere

S. Tsynkov[1,*]
[1] Department of Mathematics, North Carolina State University, Raleigh, NC, USA
*Email: tsynkov@math.ncsu.edu

Abstract

We analyze the effect of dispersion of radio waves in the Earth's ionosphere on the performance of spaceborne synthetic aperture radars (SARs). We describe the electromagnetic propagation in the framework of a scalar model for the transverse field subject to weak anomalous dispersion due to the cold plasma. Random contributions to the refraction index are accounted for by the Kolmogorov model of ionospheric turbulence. The ionospheric phenomena, both deterministic and random, are shown to affect the azimuthal resolution of a SAR sensor stronger than the range resolution; also, the effect of randomness appears weaker than that of the baseline dispersion. Probing on two carrier frequencies is identified as a possible venue for reducing the distortions.

1 Mathematical Model

The propagation of high frequency transverse electromagnetic waves in dilute plasma, e.g., the Earth's ionosphere, is governed by the Klein-Gordon equation:

$$\frac{\partial^2 \boldsymbol{E}_\perp}{\partial t^2} - c^2 \Delta \boldsymbol{E}_\perp + \omega_{\text{pe}}^2 \boldsymbol{E}_\perp = \boldsymbol{0}, \qquad (1)$$

where the quantity ω_{pe} is called the Langmuir frequency: $\omega_{\text{pe}} = \sqrt{\frac{4\pi e^2 N_{\text{e}}}{m_{\text{e}}}}$, and the electron number density N_{e} varies with altitude[1] and has a stochastic component due to the ionospheric turbulence.

The propagation of waves governed by equation (1) is dispersive, with the group velocity given by $v_{\text{gr}} = c\left(1 + \omega_{\text{pe}}^2/c^2k^2\right)^{-\frac{1}{2}}$. Hence, the dispersion is anomalous, i.e., it is weaker for the short waves.

Imaging of the Earth's surface by a spaceborne SAR is done as follows. The full radar antenna is a synthetic array, which means that the ground is illuminated by a sequence of pulses transmitted from different locations as the antenna mounted on the satellite travels along the orbit. The received signals that are scattered off the Earth's surface are then processed by means of the matched filtering. A typical waveform of the transmitted pulse is a linear upchirp, which enables a high range resolution, whereas the azimuthal resolution is achieved by summing up contributions from different pulses, which is what the synthetic array means. A standard approach to the analysis of synthetic antennas is based on the start-stop approximation [1].

Some additional assumptions that we employ include: scalar propagation (no polarization), no spatial dispersion and no Ohm conductivity, deterministic and dispersionless targets, two-dimensional images (as opposed to interferometric), isotropic plasma (no Faraday rotation) and isotropic turbulence, linearized scattering off the targets, geometrical optics and geometrical optics perturbations methods.

The turbulence is characterized by a Kolmogorov type spectrum for the fluctuations of N_{e}: $\widehat{V}(q) = \frac{C}{(1+q^2/q_0^2)^{11/6}}$. This spectrum is subsequently simplified by replacing the exponent $\frac{11}{6}$ by 2, which yields an exponentially decaying correlation function and basically amounts to considering only short range ionospheric phenomena.

2 Imaging

The key difficulty for SAR imaging through the Earth's ionosphere lies in the fact that to guarantee the best performance, the filter has to match the phase of the received signal. However, the dispersion in the ionosphere causes delays and dilations of signals, and standard matched filtering based on the concept of unobstructed propagation becomes deficient. There have been studies in the literature (see the review part in [2]) showing that very accurate matched filters can be built if the characteristics of the dispersive medium are known. Unfortunately, this cannot be considered the case for the Earth's ionosphere, which is a very "lively" medium with its key characteristics, such as N_{e}, changing rapidly, so that no precise values can be predicted at a given moment of time and given location.

If no phase correction at all is implemented in the matched filter that would account for dispersive propagation, then the quality of the image deteriorates. The corresponding analysis is conducted by

[1] It also depends on the geographic location, the time of the day, the time of the year, and many other factors.

computing the generalized ambiguity function of a SAR, which is essentially the image of one point target. We show that both the resolution of the radar (i.e., its ability to distinguish between close by targets) and the sharpness of the image may suffer from the ionospheric dispersion. For the carrier frequency of $1\,GHz$, which can be considered a mid-range value for SAR applications, the degradation can be between fractions of one percentage point and a few percentage points, depending on the specific characteristic of the image and on the values of other key parameters (such as duration of the pulse, its bandwidth, etc.) Typically, azimuthal resolution (along the orbit) is affected more than the range resolution (across the orbit). For lower carrier frequencies (UHF of VHF band) that can be useful for certain applications (ground penetration), the deterioration may be stronger. For higher carrier frequencies (microwave band), on the other hand, it is weaker.

Moreover, following the behavior of the electron number density N_{e}, the dispersion of radio waves in the ionosphere has both a deterministic and a stochastic component. The latter is attributed to turbulence, and if the correlation function for the turbulent fluctuations of N_{e} is exponential, we can show that the adverse effect of the ionosphere on the SAR image is still dominated by the deterministic dispersion. In doing so, our key consideration used when analyzing the statistics of waves in the ionosphere was normalization of the probability distributions for long propagation distances.

To implement the phase correction in the matched filter, one needs to know the characteristics of the dispersive medium (the ionosphere) at the time and place when the image is taken. To obtain those characteristics, we propose to probe the terrain, and hence the ionosphere, on (at least) two distinct carrier frequencies. It turns out that besides the deterioration of the image resolution and sharpness, one of the effects of a non-corrected filter happens to be the overall shift of the entire imaged scene. This shift depends on the value of the carrier frequency and also on the integral of the electron number density across the entire ionospheric layer. The latter is precisely the key parameter needed to adjust the filter.

Let us assume that there is an object or feature in the scene that can be clearly identified on the image. This object does not have to be artificial. It does not have to dominate the scene, say, by having the highest reflectivity. Its location does not have to be known ahead of time. It merely has to be something that can be fairly easily picked out and matched on different images. For example, it can be some landmark, a hilltop, a building, a road intersection, etc. This object will be shifted from its true position, and the shift will be different for different carriers. A simple system of equations can then be written and solved that would provide both the true position of the object and the unknown ionospheric quantity.

3 Discussion

We have analyzed the effect of ionospheric dispersion on the performance of spaceborne SAR sensors and proposed a remedy (multi-carrier probing) that may help reduce the distortions of the image. Certain aspects of the overall formulation that we have used may still require additional attention. A more accurate description of the ionospheric turbulence may be needed. In addition, the validity of geometrical optics may need to be further analyzed and potential benefits of employing a more comprehensive model of propagation, based on the paraxial approximation, will need to be studied. Besides, a more comprehensive physical framework may need to be considered that would account for the field polarization, the Faraday rotation, dispersion of the targets, etc. Finally, a thorough performance study of the phase-corrected filter is required along the lines of how it addresses both the deterministic and the stochastic part of the ionospheric dispersion.

See references [2], [1]

References

[1] S. V. Tsynkov. On the effect of start-stop approximation for spaceborne SAR imaging. *SIAM J. on Imaging Sciences*, 2009. To appear.

[2] S. V. Tsynkov. On SAR imaging through the Earth's ionosphere. *SIAM J. on Imaging Sciences*, 2(1):140–182, 2009.