MA 797 – Mathematics of Radar Imaging
Spring 2024

Instructor

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Course Meetings

Days: Monday and Wednesday
Time: 11:45 a.m.-1 p.m.
Campus: Main
Location: SAS Hall, room number TBD

Course Resources

To access your courses and project spaces, use WolfWare.

Course Description

Radar imaging is a mature technology with a broad range of applications. Yet in spite of its many demonstrated successes, a number of difficult issues are still outstanding, e.g., mitigation of the various distortions. Addressing these issues requires going beyond the standard engineering practices of the discipline and calls for employing an array of mathematical methods. Accordingly, the course will adopt the interpretation of radar imaging as a mathematical inverse problem. It will cover the key aspects of the construction and analysis of the pertinent mathematical models using tools from differential equations, perturbation theory, and Fourier analysis. Students will learn the main quantitative concepts of radar imaging: matched filtering, synthetic aperture, imaging operator, focusing, resolution, signal compression, and others. They will also be exposed to the fundamentals of electromagnetic wave propagation and scattering, from the standpoint of both physics and mathematics. Those include Maxwell's, d'Alembert (wave), and Helmholtz equations, geometrical optics, diffraction (Fresnel and Fraunhofer regimes), Doppler effects, the Born series and first Born approximation, Bragg scattering, and more.

The course is offered as a balanced combination of theory and applications. It will be of interest to students in applied mathematics, as well as physics, electrical engineering, and other areas of engineering and science. Graduate and senior undergraduate students are welcome.

Learning Outcomes

- By the time they complete the class, students will gain a solid understanding of mathematics behind a very useful and important modern technology.
- They will be able to choose and employ the appropriate mathematical tools for the quantitative analysis of the various radar imaging scenarios.

Additional Benefits to Students

Civilian applications of radar imaging range from geosciences to navigation to agriculture and forestry. Besides, radar imaging has many military applications. Students who take this class will be very well positioned for internships and then jobs related to the quantitative methods in remote sensing. The US Government organizations that have a keen interest in radar imaging include National Aeronautics and Space Administration (NASA), Air Force Research Laboratory (AFRL), Sandia National Laboratory, National Reconnaissance Office (NRO), United States Space Force (USSF), and many others.

Requisites and Restrictions

Some prior knowledge of topics in real analysis (MA425/426), partial differential equations (MA401), and Fourier series/integrals, as well as some basic understanding of waves, will be an advantage, although the most important concepts and techniques will be taught in the course itself. Taking the class without part of the recommended background will require an additional, more thorough reading.

Textbook

- The primary written source is the research monograph *Transionospheric Synthetic Aperture Imaging*, by M. Gilman, E. Smith, and S. Tsynkov, published by Birkhäuser, Cham, in 2017.
- Alternate/additional reading may be posted by the instructor in Moodle.

Course Schedule

For the first offering of the course, the following schedule will be adopted (which is subject to change).

1. (1 week) Introduction to synthetic aperture radar (SAR) imaging (Chapter 1)
2. (6 weeks) Conventional SAR imaging (Chapter 2)
    a. Propagation and scattering of radar signals
        (i) SAR interrogating waveforms
        (ii) The first Born approximation
    b. Radiation pattern of radar antenna
    c. Inversion of the raw radar data
        (i) Matched filter
        (ii) Synthetic aperture
        (iii) Imaging kernel
    d. The generalized ambiguity function (GAF)
        (i) Factorized representation of the GAF
        (ii) Azimuthal summation and pulse repetition frequency
        (iii) The azimuthal factor
        (iv) The range factor
        (v) Fourier interpretation of the SAR data inversion
        (vi) Doppler viewpoint for the azimuthal reconstruction
    e. Factorization error of the GAF
    f. SAR resolution
    g. Brief literature review
3. (4 weeks) The start-stop approximation (Chapter 6)
    a. The Lorentz transform and Doppler effect
    b. Inversion of the raw data
    c. The generalized ambiguity function
    d. Factorization error of the GAF
    e. Impact on SAR resolution
    f. SAR performance with no filter correction
        (i) GAF with no filter correction
        (ii) The range factor

        (iii) The azimuthal factor
        (iv) Factorization error
        (v) Image distortions for non-corrected filter
4. (4 weeks) Modeling radar targets beyond the first Born approximation (Chapter 7)
    a. A half-space model for radar targets
        (i) Incident field near the target
        (ii) The method of perturbations and separation of variables
        (iii) Reflected field in the frequency domain
        (iv) Reflected field in the time domain
    b. SAR ambiguity theory for the new scattering model
    c. Leontovich (impedance) boundary condition
    d. Rough surface scattering

For subsequent offering of the class, the material from Chapters 3, 4, and 5 may be included, which discusses the distortions of spaceborne radar images due to the Earth's ionosphere and their mitigation.

Course Structure

Homework

Homework will be assigned (approximately) biweekly and should be computer typeset for submission, preferably using LaTeX. Students must write up their own solutions. While working with other students is allowed, each student must submit their solutions individually. In every homework paper, students should indicate what additional sources (if any) they have used to prepare the solutions (e.g., whether they have worked or discussed the problems with any group members or read a particular written source). Using other sources, such as textbooks and monographs, in addition to the recommended textbook and notes taken in class, is acceptable to facilitate the learning. However, to do the homework it is not acceptable to copy solutions from books verbatim or use other students' solutions, online homework solutions from similar courses at other universities, or online discussion boards where people post problem solutions. Such practices violate the NCSU academic integrity policy.

Final Projects

There will be a final project at the end of the semester.

- Several topics will be suggested by the instructor and posted on the course webpage and in Moodle.
- Students may either choose a suggested topic or pick one of their own.
- Before starting to work on the final project, students will need to discuss with the instructor the details of what they intend to do.
- The final paper must be computer typeset, preferably in LaTeX.

LaTeX Resources

- Essential LaTeX
- Essential Mathematical LaTeX
- LaTeX 2e introduction
- LaTeX Software and Documentation
- Easy to install LaTeX for Windows
- Easy to install LaTeX for MacOS

Grading

Homework – 70%, final paper – 30%.

Grades are based on the following scale: A+: > 97%, A: 96.99 - 93% A-: 92.99 - 90%, B+: 89.99 - 87%, B: 86.99 - 83%, B-: 82.99 - 80%, C+: 79.99 - 77%, C: 76.99 - 73%, C-: 72.99 - 70.00%, D+: 69.99 - 67 %, D: 66.99 - 63%, D- 62.99 - 60.00%, F: <60%.

Requirements for Auditors (AU): Information about and requirements for auditing a course can be found at REG 02.20.04.

Policies on Incomplete Grades: If an extended deadline is not authorized by the Graduate School, an unfinished, incomplete grade will automatically change to an F after either (a) the end of the next regular semester in which the student is enrolled (not including summer sessions), or (b) by the end of 12 months if the student is not enrolled, whichever is shorter. Incompletes that change to F will count as an attempted course on transcripts. The burden of fulfilling an incomplete grade is the responsibility of the student. The university policy on incomplete grades is located at REG 02.50.03. Additional information about incomplete grades for graduate students can be found in the Graduate Administrative Handbook in Section 3.17.G.

Attendance: Students are expected to arrive on time, contribute to group work and class discussions, and stay until the class ends. Attendance at all meetings of the class is expected. For complete attendance and excused absence policies, please see REG 02.20.03.

Absences: NCSU policy, including what constitutes an 'Excused Absence,' is at REG 02.20.03.

Notifications

- Please check the course webpage regularly, as it will be continuously updated with announcements, any changes in the schedule, homework problems, solutions, review sheets, and other additional course materials.
- Students will also be notified in class or via email of announcements associated with this course. The email address registered with the NCSU online directory will be used for this purpose. It is the student's responsibility to maintain a valid email address and check the email sent to it.

End-of-semester class evaluation

- Schedule: Online class evaluations will be available for students to complete during the last week of class.
- Students will receive an email directing them to a website where they can log in using their Unity ID and complete evaluations.
- All evaluations are confidential; instructors will never know how any one student responded to any question, and students will never know the ratings for any instructors.
- Evaluation website: https://oirp.ncsu.edu/classeval/for-students/
- Student help desk: classeval@ncsu.edu
- More information about ClassEval: https://oirp.ncsu.edu/classeval/about-classeval/

Academic Integrity

Students are required to comply with the university policy on academic integrity found in the Code of Student Conduct (POL 11.35.01).

Academic Honesty: See POL 11.35.01 for a detailed explanation of academic honesty.

Honor Pledge: Your signature on any test or assignment indicates "I have neither given nor received unauthorized aid on this test or assignment."

Accommodations for Disabilities

Reasonable accommodations will be made for students with verifiable disabilities. To take advantage of available accommodations, students must register with the Disability Services Office at Holmes Hall, Suite 304, Campus Box 7509, 919-515-7653. For more information on NC State's policy on working with students with disabilities, please see the Academic Accommodations for Students with Disabilities Regulation at REG 02.20.01.

Non-Discrimination Policy

NC State provides equal opportunity and affirmative action efforts and prohibits all forms of unlawful discrimination, harassment, and retaliation ("Prohibited Conduct") that are based upon a person's race, color, religion, sex (including pregnancy), national origin, age (40 or older), disability, gender identity, genetic information, sexual orientation, or veteran status (individually and collectively, "Protected Status"). Additional information as to each Protected Status is included in POL 04.25.02 (Discrimination, Harassment, and Retaliation Complaint Procedure). NC State's policies and regulations covering discrimination, harassment, and retaliation may be accessed at POL 04.25.05 or the Office for Institutional Equity and Diversity. Any person who feels they have been the subject of prohibited discrimination, harassment, or retaliation should contact the Office for Equal Opportunity (OEO) at 919-515-3148.