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A Theoretical Introduction to Numerical Analysis

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Contents

Preface

This book introduces the key ideas and concepts of numerical analysis. The discussion focuses on how one can represent different mathematical models in a form that enables their efficient study by means of a computer. The material learned from this book can be applied in various contexts that require the use of numerical methods. The general methodology and principles of numerical analysis are illustrated by specific examples of the methods for real analysis, linear algebra, and differential equations. The reason for this particular selection of subjects is that these methods are proven, provide a number of well-known efficient algorithms, and are used for solving different applied problems that are often quite distinct from one another.

The contemplated readership of this book consists of beginning graduate and senior undergraduate students in mathematics, science and engineering. It may also be of interest to working scientists and engineers. The book offers a first mathematical course on the subject of numerical analysis. It is carefully structured and can be read in its entirety, as well as by selected parts. The portions of the text considered more difficult are clearly identified; they can be skipped during the first reading without creating any substantial gaps in the material studied otherwise. In particular, more difficult subjects are discussed in Sections 2.3.1 and 2.3.3, Sections 3.1.3 and 3.2.7, parts of Sections 4.2 and 9.7, Section 10.5, Section 12.2, and Chapter 14.

Hereafter, numerical analysis is interpreted as a mathematical discipline. The basic concepts, such as discretization, error, efficiency, complexity, numerical stability, consistency, convergence, and others, are explained and illustrated in different parts of the book with varying levels of depth using different subject material. Moreover, some ideas and views that are addressed, or at least touched upon in the text, may also draw the attention of more advanced readers. First and foremost, this applies to the key notion of the saturation of numerical methods by smoothness. A given method of approximation is said to be saturated by smoothness if, because of its design, it may stop short of reaching the intrinsic accuracy limit (unavoidable error) determined by the smoothness of the approximated solution and by the discretization parameters. If, conversely, the accuracy of approximation self-adjusts to the smoothness, then the method does not saturate. Examples include algebraic vs. trigonometric interpolation, Newton-Cotes vs. Gaussian quadratures, finite-difference vs. spectral methods for differential equations, etc.

Another advanced subject is an introduction to the method of difference potentials in Chapter 14. This is the first account of difference potentials in the educational literature. The method employs discrete analogues of modified Calderon's potentials and boundary projection operators. It has been successfully applied to solving a variety of direct and inverse problems in fluids, acoustics, and electromagnetism.

This book covers three semesters of instruction in the framework of a commonly used curriculum with three credit hours per semester. Three semester-long courses can be designed based on Parts I, II, and III of the book, respectively. Part I includes interpolation of functions and numerical evaluation of definite integrals. Part II covers direct and iterative solution of consistent linear systems, solution of overdetermined linear systems, and solution of nonlinear equations and systems. Part III discusses finite-difference methods for differential equations. The first chapter in this part, Chapter 9, is devoted to ordinary differential equations and serves an introductory purpose. Chapters 10, 11, and 12 cover different aspects of finite-difference approximation for both steady-state and evolution partial differential equations, including rigorous analysis of stability for initial boundary value problems and approximation of the weak solutions for nonlinear conservation laws. Alternatively, for the curricula that introduce numerical differentiation right after the interpolation of functions and quadratures, the material from Chapter 9 can be added to a course based predominantly on Part I of the book.

A rigorous mathematical style is maintained throughout the book, yet very little use is made of the apparatus of functional analysis. This approach makes the book accessible to a much broader audience than only mathematicians and mathematics majors, while not compromising any fundamentals in the field. A thorough explanation of the key ideas in the simplest possible setting is always prioritized over various technicalities and generalizations. All important mathematical results are accompanied by proofs. At the same time, a large number of examples are provided that illustrate how those results apply to the analysis of individual problems.

This book has no objective whatsoever of describing as many different methods and techniques as possible. On the contrary, it treats only a limited number of well-known methodologies, and only for the purpose of exemplifying the most fundamental concepts that unite different branches of the discipline. A number of important results are given as exercises for independent study. Altogether, many exercises supplement the core material; they range from elementary to quite challenging.

Some exercises require computer implementation of the corresponding techniques. However, no substantial emphasis is put on issues related to programming. In other words, any computer implementation serves only as an illustration of the relevant mathematical concepts and does not carry an independent learning objective. For example, it may be useful to have different iteration schemes implemented for a system of linear algebraic equations. By comparing how their convergence rates depend on the condition number, one can subsequently judge the efficiency from a mathematical standpoint. However, other efficiency issues, e.g., runtime efficiency determined by the software and/or computer platform, are not addressed as there is no direct relation between them and the mathematical analysis of numerical methods.

Likewise, no substantial emphasis is put on any specific applications. Indeed, the goal is to clearly and concisely present the key mathematical concepts pertinent to the analysis of numerical methods. This provides a foundation for the subsequent specialized training. Subjects such as computational fluid dynamics, computational acoustics, computational electromagnetism, etc., are very well addressed in the literature. Most corresponding books require some numerical background from the reader, the background of precisely the kind that the current text offers.

Acknowledgments

This book has a Russian language prototype [Rya00] that withstood two editions: in 1994 and in 2000. It serves as the main numerical analysis text at Moscow Institute for Physics and Technology. The authors are most grateful to the rector of the Institute at the time, Academician O. M. Belotserkovskii, who has influenced the original concept of this textbook.

Compared to [Rya00], the current book is completely rewritten. It accommodates the differences that exist between the Russian language culture and the English language culture of mathematics education. Moreover, the current textbook includes a very considerable amount of additional material.

When writing Part III of the book, we exploited the ideas and methods previously developed in [GR64] and [GR87].

When writing Chapter 14, we used the approach of [Rya02, Introduction].

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