In accordance with Theorem 3.6 of Section 3.1, and the discussion on page 73 that follows this theorem, we obtain the trigonometric interpolating polynomial:

$$\tilde{Q}_n(\cos\varphi, \sin\varphi, F) = \sum_{k=0}^{n} \tilde{a}_k \cos k\varphi,$$

$$\tilde{a}_0 = \frac{1}{2n}(f_0 + f_n) + \frac{1}{n}\sum_{m=1}^{n-1} f_m, \quad \tilde{a}_n = \frac{1}{2n}(f_0 + (-1)^n f_n) + \frac{1}{n}\sum_{m=1}^{n-1} f_m (-1)^m,$$

$$\tilde{a}_k = \frac{1}{n}(f_0 + (-1)^k f_n) + \frac{2}{n}\sum_{m=1}^{n-1} f_m \cos k\tilde{\varphi}_m, \qquad k = 1, 2, \dots, n-1.$$

Changing the variable to $x = \cos\varphi$ and denoting $\tilde{Q}_n(\cos\varphi, \sin\varphi, F) = \tilde{P}_n(x, f)$, we have:

$$\tilde{P}_n(x, f) = \sum_{k=0}^{n} \tilde{a}_k T_k(x),$$

$$\tilde{a}_0 = \frac{1}{2n}(f_0 + f_n) + \frac{1}{n}\sum_{m=1}^{n-1} f_m, \qquad \tilde{a}_n = \frac{1}{2n}(f_0 + (-1)^n f_n),$$

$$\tilde{a}_k = \frac{1}{n}(f_0 + (-1)^k f_n) + \frac{2}{n}\sum_{m=1}^{n-1} f_m T_k(\tilde{x}_m), \qquad k = 1, 2, \dots, n-1.$$

Similarly to the polynomial $P_n(x, f)$ of (3.62), the algebraic interpolating polynomial $\tilde{P}_n(x, f)$ built on the grid:

$$\tilde{x}_m = \cos\tilde{\varphi}_m = \cos\frac{\pi}{n}m, \qquad m = 0, 1, \dots, n, \tag{3.71}$$

also inherits the two foremost advantageous properties from the trigonometric interpolating polynomial $\tilde{Q}_n(\cos\varphi, \sin\varphi, F)$. They are the slow growth of the Lebesgue constants as $n$ increases (that translates into the numerical stability with respect to the perturbations of $f_m$), as well convergence with the rate that automatically takes into account the smoothness of $f(x)$, i.e., no susceptibility to saturation.

Finally, we notice that the Chebyshev polynomial $T_n(x)$ reaches its extreme values on the interval $-1 \le x \le 1$ precisely at the interpolation nodes $\tilde{x}_m$ of (3.71): $T_n(\tilde{x}_m) = \cos\pi m = (-1)^m$, $m = 0, 1, \dots, n$. In the literature, the grid nodes $\tilde{x}_m$ of (3.71) are known as the Chebyshev-Gauss-Lobatto nodes or simply the Gauss-Lobatto nodes.

### 3.2.7 More on the Lebesgue Constants and Convergence of Interpolants

In this section, we discuss the problem of interpolation from the general perspective of approximation of functions by polynomials. Our considerations, in a substantially abridged form, follow those of [LG95], see also [Bab86]. We quote many of the fundamental results without a proof (the theorems of Jackson, Weierstrass, Faber-Bernstein, and Bernstein). The justification of these results, along with a broader and more comprehensive account of the subject, can