For an even grid function, $f_m = f_{-m}$, formulae (3.51)–(3.54) transform into:

$$
\begin{aligned}
\tilde{a}_0 &= \frac{1}{N}(f_0 + f_n) + \frac{2}{N}\sum_{m=1}^{n-1} f_m, \\
\tilde{a}_k &= \frac{2}{N}(f_0 + (-1)^k f_n) + \frac{4}{N}\sum_{m=1}^{n-1} f_m \cos\frac{2\pi km}{N}, \qquad k = 1, 2, \dots, n-1, \\
\tilde{a}_n &= \frac{1}{N}(f_0 + (-1)^n f_n) + \frac{2}{N}\sum_{m=1}^{n-1} f_m (-1)^m \\
\tilde{b}_k &= 0, \qquad k = 1, 2, \dots, n-1,
\end{aligned}
$$

and the polynomial (3.50) reduces to:

$$
\tilde{Q}_n\left(\cos\frac{2\pi}{L}x, \sin\frac{2\pi}{L}x, f\right) = \sum_{k=0}^{n} \tilde{a}_k \cos\frac{2\pi k}{L}x.
$$

Note that the arguments which are very similar to those used when proving the key properties of the trigonometric interpolating polynomial $Q_n\left(\cos\frac{2\pi}{L}x, \sin\frac{2\pi}{L}x, f\right)$ in Theorems 3.4 and 3.5, also apply to the polynomial $\tilde{Q}_n\left(\cos\frac{2\pi}{L}x, \sin\frac{2\pi}{L}x, f\right)$ defined by formulae (3.50)–(3.54). Namely, this polynomial has slowly growing Lebesgue constants and as such, is basically stable with respect to the perturbations of the grid function f_m. Moreover, it converges to the interpolated function $f(x)$ as $n \longrightarrow \infty$ with the rate determined by the smoothness of $f(x)$, i.e., there is no saturation.

REMARK 3.1 If the interpolated function $f(x)$ has derivatives of all orders, then the rate of convergence of the trigonometric interpolating polynomials to $f(x)$ will be faster than any inverse power of n. In the literature, this type of convergence is often referred to as *spectral.* $\Box$

3.2 Interpolation of Functions on an Interval. Relation between Algebraic and Trigonometric Interpolation

Let $f = f(x)$ be defined on the interval $-1 \le x \le 1$, and let it have there a bounded derivative of order $r+1$. We have chosen this specific interval $-1 \le x \le 1$ as the domain of $f(x)$, rather than an arbitrary interval $a \le x \le b$, for the only reason of simplicity and convenience. Indeed, the transformation $x = \frac{a+b}{2} + t\frac{b-a}{2}$ renders a transition from the function $f(x)$ defined on an arbitrary interval $a \le x \le b$ to the function $F(t) \equiv f\left(\frac{a+b}{2} + t\frac{b-a}{2}\right)$ defined on the interval $-1 \le t \le 1$.

3.2.1 Periodization

According to Theorem 3.5 of Section 3.1, trigonometric interpolation is only suitable for the reconstruction of smooth periodic functions from their tables of values.