equations that governs low speed flows of incompressible viscous fluid, the Maxwell system of equations for time-harmonic electromagnetic fields, the linearized Euler equations for time-harmonic acoustics, as well as for the solution of some other equations and systems, see, e.g., [Poz02]. An obvious advantage of using boundary integral equations and the method of boundary elements compared, say, with the method of finite differences, is the reduction of the geometric dimension by one. Another clear advantage of the boundary integral equations is that they apply to boundaries of irregular shape and automatically take into account the boundary conditions, as well as the conditions at infinity (if any). Moreover, the use of integral equations sometimes facilitates the construction of numerical algorithms that do not get saturated by smoothness, i.e., that automatically take into account the regularity of the data and of the solution and adjust their accuracy accordingly, see, e.g., [Bab86], [Bel89,BK01].

The principal limitation of the method of boundary elements is that in order to directly employ the apparatus of classical potentials for the numerical solution of boundary value problems, one needs a convenient representation for the kernels of the corresponding integral equations. Otherwise, no efficient discretization of these equations can be constructed. The kernels, in their own turn, are expressed through fundamental solutions, and the latter admit a simple closed form representation only for some particular classes of equations (and systems) with constant coefficients. Linear differential equations with variable coefficients already cannot be easily handled by the boundary element method.

Another limitation manifests itself even when the fundamental solution is known and can be represented by means of a simple formula. In this case, reduction of a boundary value problem to an equivalent integral equation may still encounter difficulties because the corresponding sets of solutions are not necessarily the same. We have illustrated this phenomenon in Section 13.1 using the example of an exterior Dirichlet problem for the Laplace equation and giving an interpretation in terms of a resonance of the complimentary (interior) Neumann problem.

The most serious practical disadvantage of the boundary element method is the presence of singular integral kernels. Immediately at the boundary points, the kernel singularity can usually be handled analytically, and the fields remain bounded as long as the surfaces are smooth. However, for points in the vicinity of a surface, the evaluation of the integral is problematic, as analytical expressions are usually unavailable and numerical quadratures require extreme care.

The treatment of the boundary conditions by the boundary element method is relatively narrow. Care must be exercised, on a case-by-case basis, in the choice of the equivalent boundary sources, so that the resulting Fredholm equation is of the second kind, which is well-posed [see equations (13.6)] rather than of the first kind. Moreover, mixed (Dirichlet/Neumann, etc.) or less standard (Robin, etc.) boundary conditions require special development.

Finally, the discretization by means of boundary elements usually yields a full matrix, in contrast with the sparse finite difference or finite element matrices. This disadvantage, however, is largely alleviated by the significant progress in the development of the fast multipole method [GR87,CGR99].