where $r = \tau/h$. Form (10.140) suggests that the marching procedure for scheme (10.139) can be interpreted as consecutive computation of the grid functions:

$$u^0, u^1, \dots, u^p, \dots, u^{[T/\tau]},$$

defined on identical one-dimensional grids $m = 0, 1, \dots, M$ that can all be identified with one and the same grid. Accordingly, the functions u^p, $p = 0, 1, \dots, [T/\tau]$, can be considered elements of the linear space U'_h of functions $u = \{u_0, u_1, \dots, u_M\}$ defined on the grid $m = 0, 1, \dots, M$. We will equip this linear space with the norm, e.g.,

$$\|u\|_{U'_h} = \max_{0 \le m \le M} |u_m| \qquad \text{or} \qquad \|u\|_{U'_h} = \left[h \sum_{m=0}^{M} |u_m|^2 \right]^{\frac{1}{2}}.$$

We also recall that in the definitions of stability (Section 10.1.3) and convergence (Section 10.1.1) we employ the norm $\|u^{(h)}\|_{U_h}$ of the finite-difference solution $u^{(h)}$ on the entire two-dimensional grid. Hereafter, we will only be using norms that explicitly take into account the layered structure of the solution, namely, those that satisfy the equality:

$$\|u^{(h)}\|_{U_h} = \max_{0 \le p \le [T/\tau]} \|u^p\|_{U'_h}.$$

Having introduced the linear normed space U'_h, we can represent any evolution scheme, in particular, scheme (10.139), in *the canonical form:*

$$\begin{aligned} &u^{p+1} = \boldsymbol{R}_h u^p + \tau \rho^p, \\ &u^0 \ \text{ is given.} \end{aligned} \tag{10.141}$$

In formula (10.141), $\boldsymbol{R}_h : U'_h \longmapsto U'_h$ is the transition operator between the consecutive time levels, and $\rho^p \in U'_h$. If we denote $v^{p+1} = \boldsymbol{R}_h u^p$, then formula (10.140) yields:

$$v_m^{p+1} = (1-r)u_m^p + r u_{m+1}^p, \quad m = 0, 1, \dots M-1. \tag{10.142a}$$

As far as the last component $m = M$ of the vector v^{p+1}, a certain flexibility exists in the definition of the operator $\boldsymbol{R}_h$ for scheme (10.139). For example, we can set:

$$v_M^{p+1} = u_M^p, \tag{10.142b}$$

which would also imply:

$$\rho_m^p = \varphi_m^p, \quad m = 0, 1, \dots M-1, \quad \text{and} \quad \rho_M^p = \frac{\chi^{p+1} - \chi^p}{\tau}, \tag{10.142c}$$

in order to satisfy the first equality of (10.141).

In general, the canonical form (10.141) for a given evolution scheme is not unique. For scheme (10.139), we could have chosen $v_M^{p+1} = 0$ instead of $v_M^{p+1} = u_M^p$ in formula (10.142b), which would have also implied $\rho_M^p = \frac{\chi^{p+1}}{\tau}$ in formula (10.142c). However, when building the operator $\boldsymbol{R}_h$, we need to make sure that the following