negative real number that can be interpreted as length of the corresponding vector and quantifies the extent of its deviation from the identical zero on the grid.

There are many different ways one can define an appropriate norm for the grid function. For example, the least upper bound of all its absolute values at the grid nodes is a norm that is called the maximum norm:

$$\|u^{(h)}\|_{U_h} = \sup_n |u_n| = \max_n |u_n|. \tag{9.10}$$

If $u^{(h)}$ is a vector function, say, $u^{(h)} = \begin{bmatrix} v_n \\ w_n \end{bmatrix}$, $n = 0, 1, \dots, N$, as in scheme (9.8), then the norm similar to (9.10) can be defined as the least upper bound of the absolute values of both functions v_n and w_n on the corresponding grid.

If the grid D_h is uniform, i.e., if the grid functions $u^{(h)} \in U_h$ are defined at the equally spaced nodes $x_n = nh$, $h > 0$, $n = 0, 1, \dots, N$, then the following Euclidean norm is often used:

$$\|u^{(h)}\|_{U_h} = \left[h \sum_{n=0}^{N} u_n^2 \right]^{1/2}.$$

This norm is analogous to the continuous L_2 norm for the square integrable functions:

$$\|u(x)\| = \left[\int_0^1 u^2(x)dx \right]^{1/2}$$

Henceforth, we will always assume (for simplicity) that the maximum norm (9.10) is used, unless explicitly stated otherwise.

Having introduced the normed space U_h, we can now quantify the discrepancy between any two functions in this space. Let $a^{(h)} \in U_h$ and $b^{(h)} \in U_h$ be a pair of arbitrary functions defined on the grid D_h. The measure of their deviation from one another is naturally given by the norm of their difference:

$$\|a^{(h)} - b^{(h)}\|_{U_h}.$$

The latter quantification finally enables us to give an accurate definition of convergence for finite-difference schemes.

Let us denote by

$$\boldsymbol{L}_h u^{(h)} = f^{(h)} \tag{9.11}$$

the system of scalar equations to be used for approximately computing the solution of problem (9.1). In other words, solution of system (9.11) is supposed to yield an approximation to $[u]_h$, which is the discrete table of values for the continuous solution $u(x)$ of problem (9.1). Specific examples of the systems of type (9.11) are given by the difference schemes (9.6), (9.7), and (9.8) built for problems (9.2), (9.4), and (9.5), respectively. To recast scheme (9.6) in the general form (9.11) on a uniform