For the chord method, the quantity $\eta^{(p)}$ is chosen constant, i.e., independent of p:

$$\eta^{(p)} = \eta = \frac{F(b) - F(a)}{b - a}.$$

In other words, the value of the derivative $F'(\xi)$ anywhere on $[a,b]$ is approximated by the slope of the chord connecting the points $(a, F(a))$ and $(b, F(b))$ on the graph of the function $F(x)$, see Figure 8.2. In doing so, the iteration scheme is written as follows:

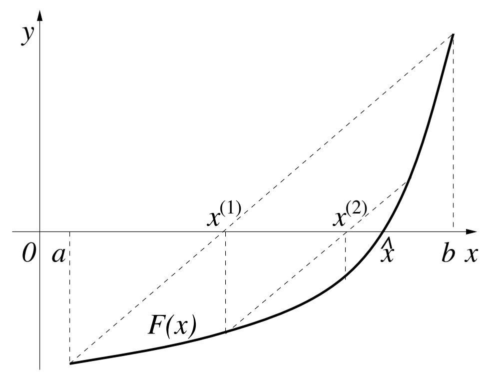

FIGURE 8.2: The chord method.

$$x^{(p+1)} = x^{(p)} - \frac{b-a}{F(b) - F(a)} F(x^{(p)}), \quad p = 0, 1, 2, \dots,$$

and to initiate it, we can formally set, say, $x^{(0)} = b$. It is possible to show (Section 8.2.1) that the sequence of iterates $x^{(p)}$ generated by the chord method converges to the root $\hat{x}$ with order $\kappa = 1$ in the sense of formula (8.1). In practice, the convergence rate of the chord method may not always be faster than that of the bisection method, for which, however, order $\kappa = 1$ cannot be guaranteed (Section 8.1.1).

8.1.3 The Secant Method

In this method, the derivative $F'(\xi) \approx \eta^{(p)}$ is approximated by the slope of the secant on the previous interval, see Figure 8.3, i.e., by the slope of the straight line that connects the points $(x^{(p)}, F(x^{(p)}))$ and $(x^{(p-1)}, F(x^{(p-1)}))$ on the graph of $F(x)$:

$$\eta^{(p)} = \frac{F(x^{(p)}) - F(x^{(p-1)})}{x^{(p)} - x^{(p-1)}}.$$

This yields the following iteration scheme:

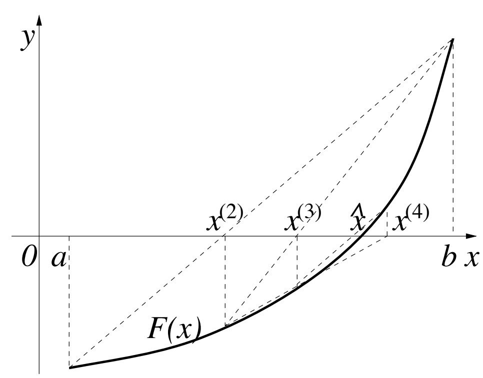

FIGURE 8.3: The secant method.

$$x^{(p+1)} = x^{(p)} - \frac{x^{(p)} - x^{(p-1)}}{F(x^{(p)}) - F(x^{(p-1)})} F(x^{(p)}), \quad p = 1, 2, 3, \dots,$$

which is started off by setting $x^{(1)} = b$ and $x^{(0)} = a$. Clearly, the first step of the secant method, $p = 1$, coincides with that of the chord method, $p = 0$, see Section 8.1.2.