Consequently, $b_{11} \neq 0$, and expression (5.53) makes sense, where

$$b'_{1j} = -\frac{b_{1j}}{b_{11}} \quad \text{for} \quad j = 2,3,\ldots,m, \quad \text{and} \quad g'_1 = \frac{g_1}{b_{11}}.$$

Moreover,

$$\sum_{j=2}^{m} |b'_{1j}| = \frac{|b_{12}| + |b_{13}| + \ldots + |b_{1m}|}{|b_{11}|},$$

hence inequality (5.54) is satisfied.

It remains to prove the last assertion of the Lemma for $m > 1$. Substituting expression (5.53) into the equation number j ($j > 1$) of system (5.51), we obtain:

$$(b_{j2} + b_{j1}b'_{12})y_2 + (b_{j3} + b_{j1}b'_{13})y_3 + \ldots (b_{jm} + b_{j1}b'_{1m})y_m = g_j - b_{j1}g'_1,$$
$$j = 2,3,\ldots,m.$$

In this system of $m-1$ equations, equation number l ($l = 1,2,\ldots,m-1$) is the equation:

$$(b_{l+1,2} + b_{l+1,1}b'_{12})y_2 + (b_{l+1,3} + b_{l+1,1}b'_{13})y_3 + \ldots + (b_{l+1,m} + b_{l+1,1}b'_{1m})y_m$$
$$= g_{l+1} - b_{l+1,1}g'_1, \qquad l = 1,2,\ldots,m-1. \tag{5.55}$$

Consequently, the entries in row number l of the matrix of system (5.55) are:

$$(b_{l+1,2} + b_{l+1,1}b'_{12}),\ (b_{l+1,3} + b_{l+1,1}b'_{13}),\ \ldots,\ (b_{l+1,m} + b_{l+1,1}b'_{1m}),$$

and the corresponding diagonal entry is: $(b_{l+1,l+1} + b_{l+1,1}b'_{1,l+1})$.

Let us show that there is a diagonal dominance of magnitude δ, i.e., that the following estimate holds:

$$|b_{l+1,l+1} + b_{l+1,1}b'_{1,l+1}| \geq \sum_{\substack{j=2,\\ j\neq l+1}}^{m} |b_{l+1,j} + b_{l+1,1}b'_{1j}| + \delta. \tag{5.56}$$

We will prove an even stronger inequality:

$$|b_{l+1,l+1}| - |b_{l+1,1}b'_{1,l+1}| \geq \sum_{\substack{j=2,\\ j\neq l+1}}^{m} \left[|b_{l+1,j}| + |b_{l+1,1}b'_{1j}|\right] + \delta,$$

which, in turn, is equivalent to the inequality:

$$|b_{l+1,l+1}| \geq \sum_{\substack{j=2,\\ j\neq l+1}}^{m} |b_{l+1,j}| + |b_{l+1,1}| \sum_{j=2}^{m} |b'_{1j}| + \delta. \tag{5.57}$$

Let us replace the quantity $\sum\limits_{j=2}^{m} |b'_{1j}|$ in formula (5.57) by the number 1:

$$|b_{l+1,l+1}| \geq \sum_{\substack{j=2,\\ j\neq l+1}}^{m} |b_{l+1,j}| + |b_{l+1,1}| + \delta = \sum_{\substack{j=1,\\ j\neq l+1}}^{m} |b_{l+1,j}| + \delta. \tag{5.58}$$