approximate solution of such a system and its exact solution, which is another key object of our study. In this section, we will therefore provide some background about linear spaces and related concepts. The account of the material below is inevitably brief and sketchy, and we refer the reader to the fundamental treatises on linear algebra, such as [Gan59] or [HJ85], for a more comprehensive treatment.

We recall that the space $\mathbb{L}$ of the elements $\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\ldots$ is called linear if for any two elements $\boldsymbol{x},\boldsymbol{y}\in\mathbb{L}$ their *sum* $\boldsymbol{x}+\boldsymbol{y}$ is uniquely defined in $\mathbb{L}$, so that the operation of *addition* satisfies the following properties:

1. $\boldsymbol{x}+\boldsymbol{y}=\boldsymbol{y}+\boldsymbol{x}$ (commutativity);
2. $\boldsymbol{x}+(\boldsymbol{y}+\boldsymbol{z})=(\boldsymbol{x}+\boldsymbol{y})+\boldsymbol{z}$ (associativity);
3. There is a special element $\boldsymbol{0}\in\mathbb{L}$, such that $\forall\boldsymbol{x}\in\mathbb{L}:\ \boldsymbol{x}+\boldsymbol{0}=\boldsymbol{x}$ (existence of zero);
4. For any $\boldsymbol{x}\in\mathbb{L}$ there is another element denoted $-\boldsymbol{x}$, such that $\boldsymbol{x}+(-\boldsymbol{x})=\boldsymbol{0}$ (existence of an opposite element, or inverse).

Besides, for any $\boldsymbol{x}\in\mathbb{L}$ and any scalar number α the *product* $\alpha\boldsymbol{x}$ is uniquely defined in $\mathbb{L}$ so that the operation of *multiplication by a scalar* satisfies the following properties:

1. $\alpha(\beta\boldsymbol{x})=(\alpha\beta)\boldsymbol{x}$;
2. $1\cdot\boldsymbol{x}=\boldsymbol{x}$;
3. $(\alpha+\beta)\boldsymbol{x}=\alpha\boldsymbol{x}+\beta\boldsymbol{x}$;
4. $\alpha(\boldsymbol{x}+\boldsymbol{y})=\alpha\boldsymbol{x}+\alpha\boldsymbol{y}$.

Normally, the field of scalars associated with a given linear space $\mathbb{L}$ can be either the field of all real numbers: $\alpha,\beta,\ldots\in\mathbb{R}$, or the field of all complex numbers: $\alpha,\beta,\ldots\in\mathbb{C}$. Accordingly we refer to $\mathbb{L}$ as to a real linear space or a complex linear space, respectively.

A set of elements $z_1,z_2,\ldots,z_n\in\mathbb{L}$ is called *linearly independent* if the equality $\alpha_1z_1+\alpha_2z_2+\ldots+\alpha_nz_n=\boldsymbol{0}$ necessarily implies that $\alpha_1=\alpha_2=\ldots=\alpha_n=0$. Otherwise, the set is called *linearly dependent*. The maximum number of elements in a linearly independent set in $\mathbb{L}$ is called the *dimension* of the space. If the dimension is finite and equal to a positive integer n, then the linear space $\mathbb{L}$ is also referred to as an n-dimensional *vector* space. An example is the space of all finite sequences (vectors) composed of n real numbers called components; this space is denoted by $\mathbb{R}^n$. Similarly, the space of all complex vectors with n components is denoted by $\mathbb{C}^n$. Any linearly independent set of n vectors in an n-dimensional space forms a *basis*.

To quantify the notion of the error for an approximate solution of a linear system, say in the space $\mathbb{R}^n$ or in the space $\mathbb{C}^n$, we need to be able to measure the length of a vector. In linear algebra, one normally introduces the *norm* of a vector for that purpose. Hence let us recall the definition of a normed linear space, as well as that of the norm of a linear operator acting in this space.