

vector multiplication. The book contains five chapters, which correspond to the five different lectures given during the summer school:

1. “Structured Matrix Problems from Tensors” by Charlie F. Van Loan;
2. “Matrix Structures in Queuing Models” by Dario A. Bini;
3. “Matrices with Hierarchical Low-rank Structure” by Jonas Ballani and Daniel Kressner;
4. “Localization in Matrix Computations: Theory and Application” by Michele Benzi;
5. “Groups and Symmetries in Numerical Linear Algebra” by Hans Z. Munthe-Kaas.

Each lecture is self-contained and, consequently, the chapters vary noticeably in terms of presentation and length, ranging from 50 to 107 pages each.

As a young numerical analyst, I gladly accepted the opportunity to review this book, hoping it would fill some important gaps in my understanding of structured matrix computations. I was not disappointed. My strategy for study was to first read quickly through an entire chapter, then selectively revisit any specific sections when I wanted more detail. This seems the natural way to read such a collection of lectures, where the specific topics and the presentation are distinct. In the end it felt more like I was reading five books instead of one.

In the first four chapters I found myself in familiar territory. The prevailing topics are specific instances of matrix or tensor structure—Toeplitz, banded, sparse, data-sparse, hierarchical, low-rank, etc.—and corresponding algorithms for factoring, computing matrix-vector products, and inverting efficiently by exploiting said structure. Before reading this book I had vague notions of how many of these algorithms worked. Now, after several months of casual study I find myself on a firmer foundation. One topic I was hoping to find more of in these chapters is efficient methods for computing eigenvalues of structured matrices. Maybe they’ll need to organize a second summer school.

Unlike the first four chapters, in the last chapter I was out of my comfort zone. This

was not the fault of the author, nor of the editors, but of this particular reader. One of the principal goals of this chapter is to develop the fast Fourier transform using group theory, obviously of great practical significance. I openly admit my knowledge (or lack of knowledge) of algebra put me at a serious disadvantage as I tried to learn this material. I think the author anticipated that there might be other numerical analysts like me who would need a bit of help, and therefore he has included a long introduction building up the basic material. I found this very helpful.

As the title suggests, this book also includes many motivating applications where structured matrices occur naturally. In many cases, the authors illustrate how the structure in the matrix or tensor is a direct consequence of structure present in the application. For example, localization in PDEs results in localization in matrix discretizations. Chapter 4 was especially interesting to me in this regard, as it highlighted some important consequences of approximation theory for computing functions of structured matrices, two of my favorite topics.

In summary, I consider *Exploiting Hidden Structures in Matrix Computations: Algorithms and Applications* a worthwhile read for anyone who wants a deeper understanding of the current algorithms used in structured matrix computations, as well as a good sampling of the applications where they naturally occur. I personally plan to keep it within easy reach.

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**Transionospheric Synthetic Aperture Imaging.** By Mikhail Gilman, Erick Smith, and Semyon Tsynkov. Birkhäuser, Boston, 2017. \$129.00. xxiv+458 pp., hardcover. ISBN 978-3-319-52125-1.

Synthetic Aperture Radar (SAR) systems typically use a moving antenna that transmits pulses of electromagnetic waves toward the ground, and to the measurements of the reflected signals they apply mathematical algorithms to form beautiful and

valuable images of our planet. Many examples of such images can be found on the websites of the Jet Propulsion Laboratory and the Sandia National Laboratory. Images from space-borne SAR systems provide unprecedented information about our earth, including information about land use, changes in the ice cover, and displacement due to earthquakes, landslides, aquifer depletion, or magma swelling beneath a volcano. Putting imaging systems on orbiting satellites is desirable because these systems make repeat passes regularly for many years and therefore can be used to monitor long-term changes.

SAR involves much sophisticated mathematics; indeed, one could say that mathematics is the key technology that makes these systems possible. Roughly speaking, SAR systems work as follows. First, one generates a waveform that is then sent to the high-power (generally nonlinear) amplifiers. This waveform must be constrained to have a constant amplitude so that the amplifiers will operate most efficiently, and after amplification, it must stay within spectral constraints. The amplified signal is then sent to the antenna. How the associated electromagnetic wave radiates from the antenna, and how the resulting wave propagates and scatters, can be modeled by Maxwell's equations, which are a hyperbolic, time-varying system of partial differential equations. In the case when the waves propagate through complex media such as foliage, soil, sea ice, or the ionosphere, Maxwell's equations become stochastic. The scattered waves, together with waves from sources of interference, impinge on the antenna and induce currents, whose (noisy) measurements constitute the received signal. The part of the signal due to scattering of the transmitted wave is typically much smaller than the noise; an initial challenge is how to detect this signal. Pulling the desired signal out of the noise is a topic in the field of statistical signal processing, and it is typically done with a certain (mathematical) filtering operation. A favorable outcome depends on the initial waveform; thus, the waveform and receiver filtering procedure should be designed together, and the design process often involves coding theory, time-frequency analysis, and optimization.

From the signals, one uses mathematical processing to form images. Essentially, image formation involves solving a (non-linear) inverse problem for Maxwell's equations. Typically, linearizing approximations are used that enable SAR image formation to be done by algorithms that are similar to those used in x-ray tomography. Some of these (linear) algorithms make use of microlocal analysis and the theory of Fourier integral operators. The linearizing approximations, however, can result in image artifacts in some cases. Mathematical analysis of the images, in turn, feeds back to the design of SAR systems, including the size, shape, orientation, and flight trajectory of the antenna, and the waveforms that are transmitted.

The study of SAR imaging involves a wealth of unsolved problems, including, for example, how to determine the antenna position from the data, how to avoid linearizing approximations, how to extract three-dimensional shape information about the (possibly moving) objects in the scene, and how to form SAR images in the presence of interference from the ionosphere.

SAR has been well studied by the engineering community, but, as this book shows, there is much to be gained by addressing the subject from the point of view of modern applied mathematics. Indeed, one of the difficulties faced by mathematicians in getting into the field is that most of the existing literature is aimed at an engineering audience and, from a mathematician's point of view, starts somewhere in the middle of the subject rather than at the beginning, which mathematicians consider to be the underlying partial differential equations. This book, written by and for mathematicians, focuses on the problem of forming SAR images in the presence of interference from the ionosphere.

Most of the existing space-borne SAR systems operate at frequencies above a gigahertz because these high-frequency waves do not interact much with the ionosphere. These high-frequency waves, however, are scattered mainly by the uppermost surface the waves encounter, which, in forested areas, is the top of the foliage canopy. To penetrate into a forest requires lower frequencies. Such lower frequencies can also

penetrate dry sand and fresh-water ice, and can therefore reveal, for example, ancient roads covered by desert sand, the internal structure of glaciers, and the ruins of Mayan cities hidden in the jungle. Consequently, there is interest in putting low-frequency SAR systems into orbit, and such systems require imaging algorithms that can compensate for ionospheric interference.

The ionosphere, moreover, is an extremely complicated medium, composed of multiple layers that change structure from day to night, that are subject to disturbances from lightning and the sun, and that support traveling waves. The electromagnetic waves transmitted by SAR systems are refracted by the ionosphere, and the direction of the transmitted electric field undergoes what is known as Faraday rotation.

The first half of this book addresses SAR imaging, the ionosphere, and modification to standard SAR to account for the presence of the ionosphere. The initial chapters of the book present systematic, clear explanations of the mathematics of SAR imaging and of mathematical models for the ionosphere. The book proceeds to develop some mathematical approaches for modifying the SAR imaging process to compensate for increasingly complex models of the ionosphere. One of the challenges, for example, is that key ionospheric parameters are unknown; the book proposes a practical method to obtain the most important of these, the Total Electron Count (TEC). There is also a clear discussion of the Faraday rotation and how to identify it and correct for it. Some of this work (especially Chapters 3–5) may turn out to be very important for practical systems, and having it all collected together, in a form accessible to readers with a mathematical point of view, is very valuable.

Really, the discussion of SAR imaging through the ionosphere ends with Chapter 5, and the following three chapters address an assortment of minor issues connected to SAR. The first, the validity of the start-stop approximation (Chapter 6), has long been settled to the satisfaction of the engineering community, but it is nice to have a careful mathematical treatment. Chapters 7 and 8 deal with models for scattering from a flat plane with varying properties; these are currently mainly of academic

interest, but may represent a first step toward more realistic modeling.

The final chapter (Chapter 9) discusses a variety of unsolved problems and open questions, with suggestions for approaches for addressing them.

In general, the book reads very well, is very clear, and contains amazingly few typographical errors. This book will be very valuable to mathematicians, physicists, and engineers interested in SAR.

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**Niche Hierarchy: Structure, Organization and Assembly in Natural Systems.**

By George Sugihara. J. Ross Publishing, Plantation, FL, 2017. \$124.95. 208 pp, hardcover. ISBN 978-1-60427-128-7.

Many of the most aesthetically pleasing works in applied mathematics use familiar ideas in a surprising way to discover something that traditional approaches fail to see. This slim book, a collection of essays that comprise the author's Ph.D. thesis from 1982, is an excellent example. The scientific questions under consideration come from ecology, though would be just as at home in economics or any discipline that studies complex systems: how does structure in the collection of available resources give rise to structure in the system of consumers, and what effect does it have on the dynamics of their interactions? In pursuit of answers, the author employs a collection of techniques that would be familiar in the modern applied topology community but are well ahead of the curve in their appearance here. The exposition of both the science and the mathematics is clear and engaging, though details in the latter case are often omitted, making the text accessible to a less technical audience.

The book opens with a brief introductory chapter, which begins with the amusing observation that the sequence in which scientific discoveries are made is often poorly aligned with the way in which the studied systems are best understood analytically.

In Chapter 2, the author describes the data under consideration—observed re-