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Spectral Performance of Nitsche's Method

Relaxing admissibility requirements (such as kinematic boundary and interface conditions) provides added flexibility in computation by accommodating non-conforming meshes (unaligned with geometric features) and non-interpolatory approximations. Such formulations that are based on Nitsche's approach to enforce surface constraints weakly, which shares features with stabilized methods, combine conceptual simplicity and computational efficiency with robust performance. The basic workings of the method are well understood, in terms of a bound on the parameter. However, its spectral behavior has not been explored in depth.

The dimension of the solution space in Nitsche's method is larger than in the corresponding standard formulation. Consequently, in addition to the physical eigenpairs which approximate the exact ones, as in the standard formulation, Nitsche's method gives rise to mesh-dependent complementary pairs associated with weak enforcement of boundary or interface constraints. The dependence of the eigenvalues on the Nitsche parameter is related to a boundary quotient of the eigenfunctions (reminiscent of the Rayleigh quotient). The boundary quotient proves to be useful for separating the two types of solutions.

Numerical studies show that, for the most part, the complementary values exhibit essentially linear growth with the parameter, whereas the eigenvalues are virtually constant, in line with the values of the corresponding boundary quotients. This behavior sheds light on the role of the stabilization parameter in attaining coercivity for the Nitsche method. Veering behavior, typical of parameterized systems, is observed, but does not impair the performance of the method. The spectrum of a reduced system obtained by algebraic elimination contains only physical eigenpairs, and is free of veering. The favorable features of the reduced system warrant its use in the solution of boundary-value problems.

To date, incompatible discretizations are rarely used for eigenvalue problems in engineering applications, possibly due to the potential presence of the mesh-dependent complementary pairs. Removing the added degrees of freedom on the boundaries addressed by the Nitsche approach by Irons-Guyan reduction is a relatively inexpensive procedure that preserves the essential structure of the underlying formulation, and yields a system that contains only eigenpairs. As such, the spectrum is virtually insensitive to stabilization beyond the threshold required for coercivity, preserving the conditioning of the standard formulation. Being free of the complementary eigenpairs, the Irons-Guyan reduced Nitsche formulation of the eigenvalue problem may be solved by any conventional eigenvalue solver. This procedure facilitates the use of Nitsche's method for formulating eigenvalue problems, and justifies its use in explicit dynamics.